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The Use of Manipulative Materials in Teaching of Arithmetic in Grade Three.

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THE USE OF MANIPULATIVE MATERIALS IN TEACHING ARITHMETIC IN GRADE THREE

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ABSTRACT

In an application of the laboratory method to the teaching of arithmetic during the past decade, there has been a marked increase in the use of manipulative materials. These are concrete devices, used as teaching or learning aids, which can be handled and moved about to represent numbers as groups and to dramatize the processes as groups in action. Theoretical and philosophical writings by educational authorities have endorsed the use of such materials, but there has been a rather complete lack of any scientific study to determine their real value.

In recognition of the need for evaluation of this growing practice, this study was conducted to determine the effectiveness of manipulative materials by comparing the growth in arithmetic achievement of two groups of third grade children in the same school, one group using pictorial and symbolic materials only and the other group using manipulative materials in addition to the pictorial and symbolic materials.

In September, 1953, two groups of third grade pupils at Myrtle Place (Public) School, in Lafayette, Louisiana, were tested for intelligence and arithmetic achievement. The two groups were then equated as nearly as possible according to basic skills in arithmetic, mental ability,

age, sex, and number. During the following semester the author taught both groups of children. She had the same overall objective, that is, to teach arithmetic as an organized, systematic way of thinking. Both groups used the same textbook and workbooks. Both used the same supplementary pictorial and symbolic materials. The same materials used alike with both groups by the same teacher constituted the control factor in the study. The additional use of certain manipulative materials with the experimental group provided the variable factor in the study.

In December tests were administered to determine the amount of growth in arithmetic achievement of each group, and of the upper thirds, the middle thirds, and the lower thirds of both groups. The differences between the means on the tests in September and in December were used to determine the significance of growth. Although there was statistically significant growth within each group, there was no significant difference between the growth of the control group and the experimental group, or their sub-groups.

Therefore, it must be concluded that within the scope of this study, which was limited to the first half of the third grade and the arithmetic learnings usually laid out for this period in the course of study, no significant gain in growth in arithmetic achievement was obtained by the additional use of manipulative materials over the use of pictorial and symbolic materials provided by the textbook and workbooks, supplemented by other conventional pictorial and symbolic materials provided for the class.

CHAPTER I

THE PROBLEM AND DEFINITION OF TERMS USED

The use of manipulative devices in teaching arithmetic is not new. There has been increased interest in these materials, however, since World War II and educators have been quick to urge teachers to use them. The Eighteenth Yearbook of the National Council of Teachers of Mathematics on "Multi-Sensory Aids in the Teaching of Mathematics," published in 1945, is evidence of the great interest in the use of manipulative as well as pictorial and symbolic materials. In a contribution to this yearbook Edith Sifton recalls some recent developments in the use of teaching aids:

> Remember those first projectors and their glass slides, and how progressive some of us felt when we began to use them in our classrooms? Shortly, we brought movies to our students and began to talk to each other about the various kinds of "visual aids." Next, we added ear appeal, and "talkies" became in our language, "audio-visual aids." Now, we find ourselves -- teachers of mathematics -- stepping to the fore with an entire yearbook devoted to "multi-sensory aids." We are noting that children learn through other avenues than their eyes and ears -- for example, their hands.[1]

In the same article Sifton recommends the use of manipulative materials.

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[1] Edith Sifton, "Multi-Sensory Aids: Some Theory and a Few Practices," Eighteenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications, Teachers College, Columbia University, 1945), p. 1.

> Put something concrete into the hands of a child, something that will enable him to enter actively into the learning situation, and auditory, visual, oral, tactual, and muscular sensations unite in a drive that has real power in forming new thought patterns.[2]

Emphasizing the use of these aids the writer asks:

> Wouldn't *you* prefer to acquire division facts with your hands and eyes, as well as with your head? Wouldn't you understand them better, and remember them longer?[3]

THE PROBLEM

Statement of the problem. It was the purpose of this study to determine the effectiveness of manipulative materials (1) by comparing the growth in arithmetic achievement of two groups of third grade pupils in the same school, one group using symbolic and pictorial materials and the other using manipulative materials as well, and (2) by comparing the advancement in arithmetic achievement of the upper thirds, middle thirds, and lower thirds of each group.

Importance of the study. No other scientific study has been made to determine the degree to which manipulative devices affect learning in arithmetic.

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[2] *Ibid*., p. 4.

[3] *Ibid*., p. 7.

DEFINITION OF TERMS USED

Manipulative materials. Those devices, used as teaching aids, which a child can handle and move about are known throughout this study as "manipulative materials." Some of these, such as clocks, cartons of eggs, and coins, have real social significance and are used in social situations. Others are designed specifically to help the learner to understand some mathematical principle.

Pictorial materials. These aids include pictures, illustrations, photographs, charts, diagrams, and similar materials.

Symbolic materials. These materials are found in textbooks, workbooks, newspaper clippings, and other printed matter in which arithmetic is expressed with abstract symbols, such as words, numbers or letters.

ORGANIZATION OF THE STUDY

This study is divided into six parts:

Chapter I. The problem is defined and definitions of terms are given. An overview of the entire study is also presented.

Chapter II. The importance that educators attach to the use of manipulative materials is shown in a review of the literature. The need for a scientific study to measure the effectiveness of these materials in teaching arithmetic is stressed.

Chapter III. An analysis of the method used in equating the control and experimental groups is included with information about the pupils and an evaluation of the tests administered.

Chapter IV. Materials, methods, and selected learning activities common to both groups are described as the control factor in the experiment. The variable factor is shown to be the special methods and learning activities using manipulative materials with just the experimental group.

Chapter V. The extent of growth in arithmetic achievement is determined by intra-group comparisons of the results of tests administered in September and December. Then the growth of the two groups, as well as their sub-groups, is compared to find if there is any significant difference.

Chapter VI. After a brief resume of the study, conclusions are drawn from the intra-group and inter-group comparisons, and some proposals for further study are made.

CHAPTER II

REVIEW OF THE LITERATURE

Much has been written about the use of multi-sensory aids in the teaching of arithmetic. Some studies have been made to compare the use of the various types of materials, but there is little or no really scientific evidence to determine the effectiveness of manipulative materials in comparison to pictorial and symbolic materials.

__Relation of instructional materials to learning.__ There is no complete agreement in the literature on the use of manipulative materials -- the nature of the materials, the way in which they should be used, or their effectiveness. Educators do agree, however, on the desirability of using sensory materials to stimulate the learner to abstract his ideas and practices.

> Sensory learning is used to supply a framework for our thinking about abstractions which have nothing to do with our senses at all.
>
> . . . .
>
> Without the experience of the actual sensory perception of the results at one time, one's imagination has nothing upon which to base its creativeness . . .
>
> Instead of making our teaching a succession of memory images, we should search for every opportunity to encourage the free imagination, the speculative twist of perceptions, the courage to think and talk about what would happen __if__, the shared delight of the new idea, the explorer's enthusiasm for new physical territories

transferred to the realms of the mind . . .[1]

Most writers have an individual theory as to the use of manipulative materials. After twenty years of experimentation Stern developed manipulative materials designed "to put the number system into such concrete form that the child would see and grasp its structure."[2] She is highly critical of the use of objects whose other associations may detract from the number notions under consideration.

> It has always been the job of the educator to put abstract number relations into concrete form which is adapted to the child's interest and his mental capacities. But while we adjust our teaching to fit the inner nature of the child, we must do so without damaging the inner nature of mathematics. Modern teaching attempts have so overdone the adjusting that the arithmetic itself is camouflaged and consequently poorly learned.
>
> Objective teaching makes sense only if the objects teach what the child is to learn. Let the object come nearer the inner nature of numbers, and the child cannot fail to "see" numbers and to think in numbers. If the objects are cookies or pencils or bunnies then the child will not grasp the number relations because they seem irrelevant. . . . It is not enough to meet numbers in life situations; the interesting appearance and the practical use of the objects to be counted may distract from the basic number concepts. . . . We do not fill life situations with numbers. We fill numbers with life.[3]

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[1] Henry W. Syer, "Sensory Learning Applied to Mathematics," Twenty-First Yearbook of the National Council of Teachers of Mathematics (Washington, D. C.: The National Council of Teachers of Mathematics, 1953), pp. 108 and 115.

[2] Catherine Stern, Children Discover Arithmetic (New York: Harper and Brothers, 1949), p. xxiii.

[3] Ibid., pp. 3-4.

Wheat even avoids using the terminology "object or concrete stage" because, he says, "it suggests the study of objects rather than the study of groups."[4] He maintains that the objects are subordinate to the groups, being seperated and recombined only in order to answer questions concerning the group. The importance of the activity is the thinking the pupils do about it, the conclusions they draw.

In support of the concept that "meanings are most readily developed in experiencing concrete things,"[5] Spencer and Brydegaard suggest that the classroom for mathematics be a "learning-laboratory." However, they say:

> This does not imply that equipment or fancy gadgets in a room or things that children build make the classroom a "learning-laboratory." Rather, a classroom becomes a learning-laboratory when it produces mental and physical activity that results in experimentation; this in turn should lead to formulation of procedures and to generalizations based upon reliable and sufficient information. The materials for the laboratory are within reach of every teacher. The materials consist, for the most part, of things that children and teacher bring into the classroom for the lessons under consideration. . . .
>
> The force behind the scenes for a laboratory for learning is the classroom teacher. The teacher who senses the procedures for making the classroom a laboratory for learning is invaluable, and no material equipment can replace him. That teacher will find some of the better commercial materials helpful, but he will not

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[4] Harry Grove Wheat, How to Teach Arithmetic (Evanston: Row, Peterson and Company, 1951), pp. 326-27.

[5] Peter Lincoln Spencer and Marguerite Brydegaard, Building Mathematical Concepts (New York: Henry Holt and Company, 1952), p. 30.

be lost without them. He is the type of teacher who can teach without textbooks. On the other hand, the teacher who doesn't sense how to stimulate his learners to experiment and to formulate procedures and generalizations is not likely to do a very different job of teaching just because he has gadgets and equipment in his room.[6]

Adding to the concept of a good teacher, Wheat says:

The good teacher is constantly on the alert for opportunity to teach a lesson, and the good teacher makes use of every such opportunity, those which the systematic studies provide regularly day by day as well as those which everyday incidental happenings provide gratis. . . .

Learning like gold is where we find it. Learning like a jewel is how we shape it. Each is a complement and a supplement to the other.[7]

It appears that educators do agree that the success of a meaningful program in arithmetic depends, to a large extent, upon methods and materials of instruction. The consensus seems to be:

There is no one method or any single type of instructional materials which will suffice in all situations. The skilful teacher selects methods and materials in terms of the outcomes to be achieved and of the needs and the interests of the children. If instruction in arithmetic is to insure a steady growth in understanding number relationships a wide variety of instructional materials must be used to enrich and to supplement the learner's experience.[8]

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6 Ibid., p. 5.

7 Wheat, op. cit., p. 72.

8 Foster E. Grossnickle, Charlotte Junge, and William Metzner, "Instructional Materials for Teaching Arithmetic," Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: The University of Chicago Press, 1951), pp. 155-56.

Psychologists Cole and Bruce point out the value of the use of manipulative materials in terms of behavior:

> Whenever the field of our perception becomes too complicated to hold in mind, to manipulate in the form of symbols, and to anticipate the consequences of trying this or that, we draw diagrams and build models. . . . As the diagram of the thought model is built, new shapes, new relations are perceived, and the individual recognizes something familiar. The Gestalists would say it facilitates the coming of insight.
>
> Thus the building of the model, the drawing of the diagram -- either, in imagination or with physical materials -- is a crucial step in the human thought process. The architect thinks with his drawing board and instruments, the machine builder with his model. . . .
>
> It is this ability to build a thought model which transforms the direct, bumbling, blundering behavior of the unthinking child into the thoughtful, planful, reasoning behavior of the adult.[9]

<u>Literature on the use of manipulative, pictorial and symbolic materials</u>. The function of these types of instructional materials "is to assure growth of understanding and the ability to make generalizations about the quantitative aspects of social situations."[10] This theory seems to be commonly held by most educators, but there is a divergence of opinion regarding the relative values and the classification of the materials.

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9 Lawrence E. Cole and William F. Bruce, <u>Educational Psychology</u> (New York: World Book Company, 1950), pp. 516-18.

10 Leo J. Brueckner and Foster E. Grossnickle, <u>Making Arithmetic Meaningful</u> (Philadelphia: The John C. Winston Company, 1953), p. 533.

Morton analyzes the levels of learning through which he says that a child should pass to attain understanding of number relationships:

> In the development of most topics the first experiences should be concrete. That is, they should be sensory in character; they should deal with objects which can be seen and handled. Later experiences may be semiconcrete; they may deal with pictures of objects and with diagrams. Still later experiences will be abstract; they will deal with symbols with no objects, or pictures, or diagrams.
>
> . . . .
>
> . . . arithmetic is basically abstract. It is necessary that the pupil move from putting groups together to adding abstract numbers. This transition from the concrete to the abstract is usually best made by going through the intermediate step in which the semiconcrete kind of experience is provided. Diagrams, drawings, and the like are semiconcrete materials. Sometimes a stage part way between the concrete and the semiconcrete -- a picture stage -- is helpful.[11]

Morton points out that the teacher must supply the object stage which obviously cannot appear in the course of study. An even more important obligation of the teacher, according to Morton, is knowing when each pupil should move from one stage to the next, so as not to hinder the understanding of the slow learner nor the advance of the quick.[12]

Irene Sauble emphasizes that obligation of the teacher:

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[11] Robert Lee Morton, Teaching Children Arithmetic (Chicago: Silver Burdett Company, 1953), pp. 4 and 52.

[12] Ibid., p. 10.

While individual differences among pupils make it necessary for pupils to work on different levels, it is the obligation of the teacher to guide pupils toward higher levels of thinking. Sensing just when and how to do this is the essence of skill and artistry in teaching.[13]

She further describes three somewhat different stages in learning:

. . . the manipulation of concrete objects represents only the first stage in the development of pupils' number ideas. In the second stage of progress pupils are able to "think" certain arrangements when the objects are present only in imagination, and in the third stage, no objects are present or imagined and pupils have developed the ability to use the language of number. As pupils analyze, assemble, and compare groups of objects, the teacher needs to guide their thinking to the point that advancement will be made steadily toward a higher stage.[14]

Wheat objects to the classification that Morton and others use. He thinks that the terminology describing the four stages -- concrete, picture, semiconcrete, and abstract -- suggests objects rather than the ways in which pupils arrange them. (<u>Cf</u>. <u>ante</u>, p. 7.)

Sooner or later the physical rearrangement with objects must yield to the imagined rearrangement. . . .

We do not use the pictures because they are pictures. The pictures do not bridge the gap between so-called concrete number and abstract number. The pictures lend themselves to the imaginary combining and separation of

---

13 Irene Sauble, "Enrichment of the Arithmetic Course," <u>Sixteenth Yearbook of the National Council of Teachers of Mathematics</u> (New York: Bureau of Publications, Teachers College, Columbia University, 1941), p. 161.

14 <u>Ibid</u>., p. 163.

groups, a preliminary step to alleviate the later and more difficult step of thinking groups together and away when no objects are present.

Adding at first is conceived as "putting together." Later it is conceived as "thinking together." Subtracting at first is "taking away," and later it is "thinking away."[15]

Grossnickle, Junge, and Metzner use the term "instructional materials" to include anything which contributes to the learning process and classify the materials as follows:

> There are four kinds or classes of instructional materials: (a) real experiences, (b) manipulative materials, (c) pictorial materials, and (d) symbolic materials. This classification cannot be regarded as rigid and inflexible. The various instructional aids overlap and blend into each other. The reader must view instructional materials as existing on a continuum in which various materials appear in increasing abstraction as one proceeds from direct experiences to symbolic materials. The lowest level of quantitative thinking results from dealing with real objects, whereas the highest level results from dealing with abstract symbols.[16]

These same authors believe that different kinds of materials are needed at various stages or steps in learning about number. They also feel that these materials should be co-ordinated and synchronized so as to form a unified program of instruction in arithmetic. They state that every teacher faces the problem of deciding the kind or kinds of

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15 Wheat, op. cit., p. 328.

16 Grossnickle, Junge, and Metzner, op. cit., pp. 161-62.

materials to use with each pupil. In discussing when to use objective materials, they say:

> It is important to know if all pupils are to begin a topic or process with manipulative materials or if it is possible for some pupils to begin with symbolic materials.
>
> It is the function of good instruction in arithmetic to have a pupil operate at the highest level of difficulty at which the work is meaningful to him. It is obvious that there are pupils who develop sufficient insight into the meaning of number that the use of visual and manipulative materials is not needed to introduce a new process.[17]

Noting the value of the use of pictorial materials in problem solving, Clark, Otis, and Hatton agree:

> It would appear that some individuals learn to solve problems with very little visual imagery, while others find that a clear mental picture of the situation is very helpful in comprehending the problem and in choosing the right operation, and that lack of a clear mental picture of the situation results in inability to decide what processes to use. For such individuals it is helpful, of course, to represent situations pictorially.[18]

Gibb, in a study of children's thinking in the process of subtraction, found geometric forms more helpful in problem solving. She says:

> Children more readily understood the problems, solved them by the process of subtraction, obtained correct solutions, and worked them in less time when the problems were presented with circles (or squares) on cards.

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17 *Ibid.*, pp. 171-72.

18 John R. Clark, Arthur S. Otis, and Caroline Hatton, *Primary Arithmetic Through Experience* (Yonkers-on-Hudson, New York: World Book Company, 1939), pp. 205-06.

> They did not do as well with the problems when given the toys or other manipulative objects [whether in reality or in picture] and had most difficulty with problems presented with a written description.[19]

Rosenquist uses the terminology "concrete materials" to cover actual objects such as books, dolls, money. Like Stern, she insists that pupils can be better taught with materials "which represent the arithmetic situations included in experiences, and specifically clarify the number ideas."[20] These she calls "representative materials"; they are similar to what other writers call "semiconcrete."

> They are used to represent the number ideas and relationships of a complex experience in a simple form. They help children think of the number relationships apart from the total experiences . . . They are small objects, uninteresting in themselves, which children can use as counters. Through using these materials, number ideas and relationships can be visualized and made clear.[21]

She criticizes the use of complex materials,

> . . . such as a picture of children playing with many toys, or a play store in which the dramatic play overshadows the number ideas involved in it. Children do not learn to understand numbers through the use of materials in which the number ideas are embedded, because their attention is not attracted to the quantitative ideas and the number relationships which such materials and situations present.[22]

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19 E. Glenadine Gibb, "Take-Away Is Not Enough!" The Arithmetic Teacher, I (April 15, 1954), 10.

20 Lucy Lynde Rosenquist, Young Children Learn to Use Arithmetic (Boston: Ginn and Company, 1949), p. 68.

21 Loc. cit.

22 Ibid., pp. 68-69.

A recent experimental study by Dawson[23] challenges the validity of the four stages through which Morton and other writers insist that children move in learning the concept of number as a group. The theory, he says, was proposed by DeMay in 1935, but no data accompanied it. "No evidence exists that this hierarchy of representation has been evaluated through experimental use in the classroom,"[24] says Dawson.

Questioning the validity of DeMay's theory Dawson conducted a study, the results of which indicate that pictorial forms should not necessarily precede geometric forms. According to his conclusions:

> . . . first-graders were able to perceive groups in the simple geometric forms most easily. Responses to these forms were more mature and faster.
>
> The data indicate that complexity impeded the perception of "groupness." The critical factor in apprehension of the group was its complexity and not its "geometric" or "pictorial" form.[25]

Therefore Dawson recommends that:

> . . . increased attention should be given to the nature (complexity) of pictures used in textbooks, workbooks or otherwise to develop the concept of number as a group. The use of complicated group pictures tends

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[23] Dan T. Dawson, "Number Grouping as a Function of Complexity," The Elementary School Journal, LIV (September, 1953), 35-42.

[24] Ibid., p. 35.

[25] Ibid., p. 42.

> to produce counting, not grouping, and hence will not assist the learner in developing the ability to apprehend number groups. For the primary grades it is important to use grouping composed of relatively simple elements. It would make little difference whether the number groups were pictures of things or of geometric forms if they met the criterion of simplicity.[26]

Support is given to the use of manipulative materials by Nicholas and Troisi in a report on their exploratory study of the use of such materials in the teaching of fractions in grade six.[27] They concluded that there was no gain in computation through the use of the aids; however, a marked gain was shown in problem-solving and in understanding the nature of fractions.

This exposition of the use of manipulative materials may well be concluded with a few cautions stated by Berger after he had suggested some principles for guiding the use of learning aids, from his own experience and observation.

> The first of these is that the use of devices is not always economical or beneficial to the learning situation. Occasionally it is much easier, less time-consuming and more effective simply to <u>tell</u> what is to be communicated about a concept. Poor selection of devices, and their misuse or over-use, may actually impede the learning process or carry confusion beyond the point of repair. So one must be careful.
>
> Finally it should be pointed out that the use of learning aids is not something which is distinct from

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26 *Ibid.*, p. 41.

27 Paul Neureiter and Nicholas Troisi, "Ol' Man 'Rithmetic -- Still Maturing," *New York State Education*, XXXIX (May, 1952), 601.

all other instructional activity. Devices should be a part of instruction; they are not a substitute for it.[28]

Need for scientific study as reflected in the literature. Until the present time no scientific study to determine the effectiveness of manipulative materials has been made. Morton pointed out the lack in 1953 and qualified his recommendation of the use of concrete and semiconcrete materials, saying:

> Until we have scientific studies which evaluate the use of these materials, which we do not have as yet, the teacher should permit their use as long as it is reasonably certain that they aid the pupil to discover meaning. The test of their value lies in whether the pupil gives evidence of making a transition from the concrete aids to the abstraction whose truth they demonstrated.[29]

Grossnickle recognizes the need and proposes a study for the determination of the effectiveness of the use of manipulative materials in Chapter XV, "Needed Research on Arithmetic," in the Fiftieth Yearbook of the National Society for the Study of Education, Part II.[30] He believes such

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28 Emil J. Berger, "Principles Guiding the Use of Teacher- and Pupil-Made Learning Aids," Twenty-Second Yearbook of the National Council of Teachers of Mathematics (Washington, D. C.: The National Council of Teachers of Mathematics, 1954), pp. 169-70.

29 Morton, op. cit., pp. 53-54.

30 Foster E. Grossnickle, "Proposals for Research on Problems of Teaching and of Learning in Arithmetic," Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: The University of Chicago Press, 1951), p. 285.

a study to be significant because of the relationship of the problem to the question of teaching arithmetic meaningfully.

The danger of inaccuracy in proposing or developing a theory without experimental corroboration is cited by Dawson.[31] It has already been noted that no scientific evidence accompanied DeMay's theory of four stages of learning, in 1935. Much that was written later was based on this concept, still without testing.

Neureiter and Troisi[32] did take a step toward solving the problem in 1952; however, there were certain inherent limitations in their study.

> The grade was divided into halves, equated as much as possible by means of pre-tests . . . Naturally conclusions are somewhat tentative because the groups were relatively small and the results not confirmed by repetition.[33]

The two groups consisted of only about fifteen pupils each.[34] Neureiter and Troisi had planned to continue their study on a broader scale and with the use of better statistical methods, but their partnership had to be dissolved before the preliminary results could be confirmed.[35]

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31 Dawson, op. cit., p. 35.

32 Neureiter and Troisi, Op. cit., pp. 599-602.

33 Ibid., p. 601.

34 Personal Correspondence with the Author, letter from Paul Neureiter, February 4, 1954.

35 Loc. cit.

<u>Summary</u>. This review of the literature relating to the use of manipulative materials shows a great dearth of really controlled, experimental studies which test their effectiveness. Of the wealth of writings concerning this problem, practically all have been theoretical and philosophical. Among these writings, however, these generalizations seem to stand out:

1. Manipulative materials, along with pictorial and symbolic materials, provide multiple paths to understanding numbers and the processes.

2. The design of the teaching materials should meet the criterion of simplicity. Factors of form, shape, and pattern of arrangement should facilitate the perception of "groupness."

3. The force behind the effective use of instructional materials is the classroom teacher.

4. There is a real need for a scientific study to determine the effectiveness of the use of manipulative materials.

CHAPTER III

METHOD OF EQUATING GROUPS

Fifty-four third grade pupils at the Myrtle Place (Public) School, in Lafayette, Louisiana, were studied in this experiment. At the beginning of the semester they were given three tests, the California Test of Mental Maturity, a Stanford Arithmetic Achievement Test, and an original objective test, the results of which served as bases for forming two comparable groups. The groups were then equated as nearly as possible according to the following factors: (1) basic skills in arithmetic, (2) mental ability, (3) age, (4) sex, and (5) number. To formally check the equation of the principal factors of arithmetic skills and mental ability, the standard deviations between the means of each group for the various tests were calculated. Since both groups were small the sums of the squares of the deviations from the means were pooled to get a better estimate of the standard deviation. The following formulas[1] were used in finding critical ratios (t's) to test for any significant difference between the means of the groups:

---

[1] Henry E. Garrett, Statistics in Psychology and Education (New York: Longmans, Green and Company, 1947), p. 206.

(1) SD or $s = \sqrt{\dfrac{\Sigma(X_1 - M_1)^2 + \Sigma(X_2 - M_2)^2}{(N_1 - 1) + (N_2 - 1)}}$

(standard deviation when two small independent samples are pooled)

(2) SE_D or $s_D = s\sqrt{\dfrac{N_1 + N_2}{N_1 N_2}}$

(standard error of the difference between means in small independent samples)

(3) $t = \dfrac{D}{s_D}$

where D = difference between the means.

FACTORS EQUATED

Basic skills in arithmetic. The grade placements which the pupils made on the Stanford Achievement Test served as the chief factor in equating the groups. (See Tables XIII and XV in Appendix A.) After all papers had been scored, the test papers were arranged in descending order according to grade placements. Members for the two groups were tentatively chosen by assigning alternate papers to the control group and to the experimental group respectively. Interestingly enough, these two tentative groups actually became the control and experimental groups.

Checking on this method after the selection had been made, the mean achievement in arithmetic for each group was found. A comparison is shown in Table I.

TABLE I

COMPARISON OF THE BASIC SKILLS IN ARITHMETIC OF THE TWO GROUPS ON FORM J OF THE STANFORD ACHIEVEMENT TEST, SEPTEMBER*

Group	Number	Mean grade place-ment	Mean differ-ence	Standard devia-tion or s	Stan-dard error or s_D	Crit-ical ratio or t
Experimental	27	2.7	zero	.41	.11	zero
Control	27	2.7				

* Based on information in Tables XIII and XV, Appendix A.

It can be seen that the mean grade placement was 2.7 for each group and that the standard deviation of the means for each group was .41, indicating not only that the two groups were equal in mean arithmetic achievement, but that they were also equally variable in arithmetic achievement.

Table II gives a comparison of the arithmetic achievement scores of the two groups on the original objective test. The mean score was 97 for the control group and 100 for the experimental group; there was a difference of 3. The critical ratio for 50 degrees of freedom present in the 52 cases pooled must be 2.40 to be significant at the .01 level. The obtained critical ratio of .29 is so much less than 2.40, that the difference between the means must be considered insignificant.

TABLE II

COMPARISON OF THE BASIC SKILLS IN ARITHMETIC OF THE TWO GROUPS ON THE ORIGINAL OBJECTIVE TEST, SEPTEMBER*

Group	Number	Mean score	Mean difference	Standard deviation or s	Standard error or s_D	Critical ratio or t
Experimental	26	100	3	11.9	10.4	.29
Control	26	97				

* Based on information in Tables XIV and XVI, Appendix A.

The data presented in Tables I and II give evidence that the groups were about equal in basic skills in arithmetic.

<u>Mental ability</u>. To compare the intelligence of the two groups the mean intelligence grade placements were determined and the critical ratio calculated as shown in Table III. The mean intelligence grade placement was 3.1 for both groups. The standard deviation from the mean was 1.1 for the experimental group and 1.0 for the control group. It can be quickly seen that the two groups were closely equated in intelligence; the difference in the standard deviation suggests that the experimental group was only slightly more variable in intelligence than the control group.

TABLE III

COMPARISON OF MENTAL ABILITY OF THE TWO GROUPS ON THE CALIFORNIA TEST OF MENTAL MATURITY, SEPTEMBER*

Group	Number	Mean grade placement	Mean difference	Standard deviation or *s*	Standard error or s_D	Critical ratio or *t*
Experimental	27	3.1	zero	1.05	.28	zero
Control	27	3.1				

* Based on information in Table XII, Appendix A.

__Age.__ The mean chronological age, shown in Table IV, was eight years and three months for the control group and eight years and four months for the experimental group. The ages of the children in the experimental group ranged from seven years and eight months to nine years and three months. There was a difference within the group of one year and seven months. The youngest child in the control group was seven years and six months and the oldest was ten years. The range was two years and six months. (See Table XII, Appendix A.)

__Sex.__ Though inadvertently so, the two groups were almost evenly balanced between boys and girls. From Table IV it may be noted that there were 14 boys and 13 girls in the control group, and 13 boys and 14 girls in the experimental group.

TABLE IV

SUMMARY TABLE: COMPARISON OF THE TWO GROUPS ON THE SIX FACTORS CONSIDERED IN EQUATING THEM

Factors equated	Control group	Experi-mental group
Basic skills in arithmetic		
Mean grade placement, Stanford Test	2.7	2.7
Standard deviation	.4	.4
Mean score, original objective test	97	100
Standard deviation	23.6	24.2
Mental ability		
Mean grade placement, California Test	3.1	3.1
Standard deviation	1.0	1.1
Mean chronological age	8-8	8-4
Sex		
Boys	14	13
Girls	13	14
Number in group	27	27

Number. Originally there were thirty children in each group but due to transfers it was possible to use only twenty-seven pupils from each group in this study.

From an analysis of Table IV, page 25, it will be noticed that the groups were equal in number and nearly equal in mean chronological age and sex as well as in arithmetic skills and mental ability. Considering the small number of children, sixty, from which the experimenter drew her sample of fifty-four, the degree of homogeneity secured seems more than adequate to show that the two groups were equated.

Equation of the sub-groups of the control and experimental groups. After the two groups had been equated as a whole, a check was made to determine whether the upper thirds, middle thirds, and lower thirds of each group were equal in mental ability and arithmetic achievement. Again the groups were small and to get the best estimate of the standard deviation the samples were pooled, the upper third of the control group with the upper third of the experimental group, and so on down. With the 18 cases thus obtained there were 16 degrees of freedom; a critical ratio of 2.58 was necessary to be significant at the .01 level. From the comparison in Table V it can be seen that not one of the critical ratios was as high as 2.58; therefore the subgroups were considered equal.

TABLE V

COMPARISON OF MENTAL ABILITY AND ARITHMETIC ACHIEVEMENT BETWEEN THE SUB-GROUPS OF THE EXPERIMENTAL AND CONTROL GROUPS, SEPTEMBER#

Sub-group	Number		Mean		Mean difference	Standard deviation or s	Standard error or s_D	Critical ratio or t
	Exp.	Con.	Exp.	Con.				
California Test of Mental Maturity								
Upper thirds	9	9	3.9	3.6	.3	.80	.38	.79
Middle thirds	9	9	3.2	2.7	.5	.85	.40	1.25
Lower thirds	9	9	2.4	2.9	.5	1.18	.56	.89
Stanford Achievement Test								
Upper thirds	9	9	3.2	3.1	.1	.22	.10	1.00
Middle thirds	9	9	2.7	2.6	.1	.11	.05	2.00
Lower thirds	9	9	2.3	2.2	.1	.24	.11	.91
Original objective test								
Upper thirds	9	8[x]	124	116	8	10.54	5.16	1.55
Middle thirds	8[x]	9	96	102	6	18.17	8.90	.67
Lower thirds	9	9	80	74	6	18.42	8.66	.69

Based on information in Tables XII to XVI, Appendix A.

x One pupil missed testing.

GENERAL INFORMATION ABOUT THE PUPILS

The pupils involved in this study were rather homogeneous in their background. Most of them came from attractive homes. Their parents were interested in their achievement at school and were friendly with the teachers and eager to cooperate. The income of the fathers was above average. Most of the fathers were engaged in professional or public service work, about one-third being employed by oil companies. The children attended school regularly. Their morale was good and their attitude was one of helpfulness. On a whole the atmosphere in the home and at school was conducive to learning.

TESTS ADMINISTERED

The California Test of Mental Maturity. This test was used to measure the general intelligence because the scores could be translated into intelligence grade placements as well as Intelligence Quotients. The intelligence grade placements made for easier comparison with the grade placements on the arithmetic achievement test. The I.Q.'s were helpful in analyzing the relationship existing between each pupil's mental ability and his arithmetic achievement.

The Stanford Achievement Test, Primary Battery, Forms J and K. This particular test was chosen because it included a reasoning test as well as a computation test. The reasoning test not only measured problem-solving but also number concepts. Furthermore, the test measured skills and concepts beyond the course of study for the first semester of the third grade.

Form J was administered in September and Form K in December.

The original objective test. (Appendix B.) This test was constructed because there was no standardized test available which measured understandings of numbers or understandings of the processes to the degree that is merited by the emphasis now placed on this particular objective in the teaching of arithmetic.

The test was divided into three parts, one for each general objective. Part I measured *Concepts and Background*. Part II dealt with *Computation*, and Part III consisted of verbal *Problems*. The textbook, *Row-Peterson Arithmetic, Book Three*, was examined page by page to get a detailed list of specific objectives, and then test situations were constructed to measure these objectives.

The validity of the original test was determined by correlating its scores with those of the Stanford Achievement Test. The product moment coefficient of correlation was used. The coefficient of correlation (*r*) was calculated from the ungrouped scores of the original test and the Stanford Achievement Test, according to the formula:[2]

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \times \Sigma y^2}}$$

$$= \frac{394.15}{\sqrt{8.54 \times 28,756.58}}$$

$$= .961$$

Since perfect, positive correlation is 1.00, this coefficient of .961 is very high, indicating that the original test measured what it purported to measure, assuming that the Stanford Test itself is valid. According to Garrett, "a high correlation between a test and a criterion is evidence of validity provided the test and the criterion are both reliable."[3]

The reliability of the scores for this test was determined by the method of rational equivalence which stresses the intercorrelations of the items in the test and the correlations of items in the test as a whole. The following formula was used to compute the reliability of the homemade test:

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[2] *Ibid.*, p. 289.

[3] *Ibid.*, p. 395.

$$r_{1I} = \frac{n}{(n - 1)} \times \frac{SD^2{}_t - \Sigma pq}{SD^2{}_t}$$

(reliability coefficient of a test in terms of the difficulty and the intercorrelations of test items)

in which:

r_{1I} = reliability coefficient of the whole test

n = number of items in the test

SD_t = the standard deviation of the test scores

p = the proportion of the group answering a test item correctly

q = the proportion of the group answering a test item incorrectly[4]

$$r_{1I} = \frac{147}{146} \times \frac{558.01 - 25.14}{558.01}$$

$$= .79$$

The reliability coefficient was .79. To be significant at the .01 level with 50 degrees of freedom the coefficient need only be .354. To be significant at the .01 level means that there are 99 chances in 100 that the coefficient of correlation shows a true relationship. There is only one chance in 100 that the coefficient is the result of chance factors. Reference to Garrett's table[5] showing the

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[4] Ibid., p. 384.

[5] Ibid., p. 464.

significance of correlation coefficients based on various-sized samples indicates that at the .01 level .354 is significant for a sample containing N cases - 2, or 50 degrees of freedom (nearest ten to the 54 cases - 2 in this study). Since the coefficient need have been only .354 for this size sample, the obtained coefficient of .79 is far in excess of that and therefore is highly significant. It must be kept in mind that the significance of any coefficient of correlation depends to a large extent upon the size of the sample on which it is based and upon the representativeness of the constituents of that sample. The high coefficient of correlation means that the original objective test has internal consistency and can be depended upon to discriminate in favor of the more able pupils.

The original test was given in September and repeated in December. Since the reliability and validity of the original objective test were established, the results from this test were important in the study.

NORMALITY OF SAMPLE

The results from the tests in intelligence and in arithmetic achievement were checked to determine how nearly the 54 cases in this study approximated a normal distribution. The percents in Table VI suggest only a slight deviation from a normal distribution. In the table the percents

TABLE VI

A TEST FOR NORMALITY OF THE SAMPLE BASED ON THE COMPARISON OF THE PERCENTS OF THE 54 CASES IN BOTH GROUPS WITH THE FRACTIONAL PARTS OF THE TOTAL AREA UNDER THE NORMAL PROBABILITY CURVE, CORRESPONDING TO DISTANCES ON THE BASELINE BETWEEN THE MEAN AND SUCCESSIVE POINTS LAID OFF FROM THE MEAN IN .5 SIGMA UNITS*

Mean plus S.D. units	Percent expected	Percent on California Test	Percent on Stanford Test, Sept.
.5	19.15	16.65	18.50
1.0	34.13	29.60	25.90
1.5	43.32	33.30	37.00
2.0	47.72	44.40	44.40
2.5	49.38	46.25	44.40
3.0	49.86	46.25	44.40
Mean minus S.D. units			
3.0	49.86	51.80	46.25
2.5	49.38	51.80	46.25
2.0	47.72	49.95	46.25
1.5	43.32	46.25	44.40
1.0	34.13	37.00	27.75
.5	19.15	27.75	14.80

* Based on information in Tables XII, XIII, and XV, Appendix A.

of a normal distribution are recorded at intervals of .5 sigma units between plus and minus 3 signma units. The percents obtained from the results of the intelligence test and the achievement test both approximate very closely those of a normal distribution. To check it formally, the amount of skewness from the normal distribution was calculated according to this formula:[6]

$$Sk = \frac{(P_{90} + P_{10})}{2} - P_{50}$$

California Test of Mental Maturity:

$$Sk = \frac{(4.7 + 1.8)}{2} - 3.0$$

$$= 3.25 - 3.0$$

$$= .25$$

Stanford Achievement Test, September:

$$Sk = \frac{(3.1 + 2.2)}{2} - 2.65$$

$$= 2.65 - 2.65$$

$$= 0$$

On the California Test of Mental Maturity the scores of both groups in the sample were skewed slightly in a positive direction indicating that the scores were concentrated

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[6] *Ibid.*, p. 121.

a little below the mean. The significance of this skewness was determined by means of the formula:[7]

$$SD_{sk} = \frac{.5185D}{\sqrt{N}}$$

in which $D = (P_{90} - P_{10})$

Then $t = \frac{Sk}{SD_{sk}}$

California Test of Mental Maturity:

$$SD_{sk} = \frac{.5185 \times 2.9}{54} = .20$$

$$t = \frac{.25}{.20} = 1.25$$

With 50 degrees of freedom, the critical ratio, or t, must be 2.40 to be significant at the .01 level.[8] With this critical ratio of only 1.25 it seems quite certain that the distribution of scores on the California Test of Mental Maturity was not significantly skewed. The skewness of the scores on the Stanford Achievement Test was zero which indicated a normal distribution.

Since there was no significant skewness, it can be assumed that the 54 pupils in this study approximate a

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[7] *Ibid.*, p. 221.

[8] *Ibid.*, p. 464.

normal distribution on the basis of intelligence and of arithmetic achievement. The normality of distribution in achievement grade placements and in intelligence grade placements for the pupils studied offsets to a great extent the smallness of the sample. Furthermore, the representativeness of the sample increases the importance attached to the reliability and validity coefficients of the original objective test.

CHAPTER IV

MATERIALS, METHODS, AND LEARNING ACTIVITIES

Once the control group and the experimental group were equated, common learning experiences were planned for both groups with the use of the same two-dimensional pictorial and symbolic materials. The same materials, used alike with both groups by the same teacher, served as the control factor in the study. Additional experiences were outlined for the experimental group with the three-dimensional materials which were used exclusively with that group. It was the additional use of these materials with this group which provided the variable in the experiment.

MATERIALS COMMON TO BOTH GROUPS

The control group and the experimental group used identical symbolic and pictorial materials.

Symbolic materials.

1. Row-Peterson Arithmetic, Book Three.[1] This well-illustrated textbook was used with both groups because it was the one adopted by Lafayette Parish.

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[1] Harry Grove Wheat, Geraldine Kauffman, and Harl R. Douglass, Row-Peterson Arithmetic, Book Three (Evanston, Illinois: Row, Peterson and Company, 1952), 312 pp.

2. Workbook, Row-Peterson Arithmetic, Book Two.[2]

3. Workbook, Row-Peterson Arithmetic, Book Three.[3]

4. Discovering Arithmetic, Book 2.[4]

The variety of exercises in the three workbooks enabled the teacher to provide for the individual differences of the pupils, whose achievement grade placements ranged from 1.8 to 3.6 and whose intelligence grade placements showed an even greater range, from .7 to 5.2.[5] The second grade workbooks presented different approaches to the study of numbers, which helped to enlarge the pupils' view of the work previously studied.

Pictorial materials. Charts were selected or designed to point up number relations in a dramatic fashion. The charts provided learning experiences that supplemented those experiences provided by the textbook and workbooks. Only a few commercial materials were used. The following

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[2] Harry Grove Wheat and Margaret Leckie Wheat, Workbook, Row-Peterson Arithmetic, Book Two (Evanston, Illinois: Row, Peterson and Company, 1951), 128 pp.

[3] Harry Grove Wheat, Margaret Leckie Wheat, and Robert H. Koenker, Workbook, Row-Peterson Arithmetic, Book Three (Evanston, Illinois: Row, Peterson and Company, 1952), 128 pp.

[4] Catherine Stern, Discovering Arithmetic, Book 2, teachers' edition (Dallas: Houghton Mifflin Company, 1952), 96 + 128 pp.

[5] Reference Tables XII, XIII, and XV, Appendix A.

charts were designed by the investigator:

1. Number Names and Number Symbols. Plate I, page 43.

2. Hindu-Arabic and Chinese Number Symbols, Plate II, page 44.

3. A Configuration of 100. Plate III, page 45.

4. Component Parts of Ten. Plate V, page 47.

5. A "Moving Picture" of the Teen Numbers. Plate VI, page 48.

6. Analysis of the 11-group. Plate IX, page 52.

7. Analysis of the 12-group. Plate VIII, page 51.

8. Structure of Even and Odd Numbers in the First Decade. Plate X, page 53.

9. Structure of Even and Odd Numbers in the Second Decade. Plate XI, page 53.

10. The Fact Finding Chart. Plate XII, page 55.

11. Multiplication Pairs: 3 X 6 and 6 X 3. Plate XIV, page 58.

12. Studying Groups by Counting. Plate XV, page 60.

13. Finding a New Multiplication Fact from a Related Fact. Plate XVI, page 60.

14. Multiple Groups of Six. Plate XVII, page 61.

15. Making the Transfer from Multiplying Ones to Multiplying Tens and Ones. Plate XVIII, page 62.

METHODS COMMON TO BOTH GROUPS

The investigator taught both groups. Using the laboratory method she introduced concepts and processes with diagrams and charts supplementing the textbook and workbooks. From time to time during the semester, concepts and processes beyond the prescribed course of study were introduced in their relationship to the work at hand.

General objective. The overall objective of the investigator was to give the pupils a way of thinking, a definite, systematic way of thinking about numbers of things. In order to stimulate the pupils' thinking, experiences were provided which enabled them (1) to study numbers as groups and (2) to see the number processes as groups in action. The teacher carefully supervised each stage of the pupils' development to keep the activity a thinking activity.

Study habits of pupils. From the start the experimenter's task was twofold: (1) to teach effective methods of study through demonstrations before the pupils, and (2) to direct the pupils in using independent methods of study.

Individual differences. In studying the representations and rearrangements of number groups, the investigator worked at different levels according to the readiness of the pupils.

Varied procedures. The pupils in both sections were shown a variety of ways to study number-groups. The first part of the semester was spent in studying the number-groups to and including ten. The pupils had experience in:

1. Counting groups.

2. Comparing groups.

3. Separating and combining groups.

4. Dividing factorable groups into their component equal groups.

Decimal nature of number system. The latter part of the semester was spent in developing and using the idea of ten. The children were imbued with the principle that tens are treated the same as ones in adding, subtracting, multiplying, and dividing.

The teen numbers were studied as groups of ten and smaller groups rather than as single groups. All the teen numbers were analyzed for their component parts. The factorable ones were also divided into equal groups to pave the way for the ideas of division and multiplication.

In understanding the structure of other two-place numbers the idea of ten took on added significance. The children analyzed these numbers by separating them into tens and ones. The numerals, including zero, were seen as place holders which designated the number of units of a particular denomination.

The relation between the addition and subtraction facts of the first decade and those of higher decades was shown by focusing attention on the numbers in the same relative position in each decade, rather than by the usual method of calling attention to endings.

Preparation was made for carrying forward a group of ten in addition and multiplication, and for using a ten-group in subtraction and division.

Mastery techniques. In general the basic addition and subtraction facts were mastered by a deductive method in place of the conventional type of repetitive drill. Provision was made for practice after understanding had been established. A great effort was made to vary the practice work in order that it should be a refreshing experience.

Problem solving. Understanding numbers and the processes, and knowing the basic addition and subtraction facts, paid great dividends in the solution of problems.

SELECTED LEARNING ACTIVITIES WITH BOTH GROUPS

Concepts were introduced or enlarged through the use of charts and diagrams. Some of the concepts were approached through social situations, such as telling time, or studying coins; others were approached from a purely mathematical point of view, such as an understanding of the structure of numbers.

Appreciating our number system. It was pointed out to the pupils that number names may have been among the first words used as people began to talk. These names were necessary in speaking of animals, birds, tools -- anything with which men dealt in their daily lives.

Today the number names bear a striking similarity in the different languages.

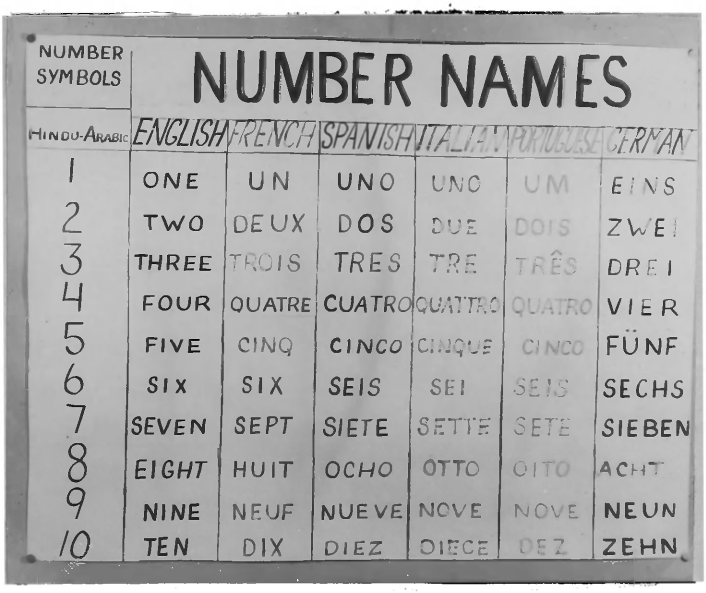

Plate I. Hindu-Arabic Numerals, a Universal Language

The children studied the names on this chart and noticed those with great similarity in the various languages, such as one, six, and eight.

An even more amazing fact was pointed out: that most countries now use the same number symbols, the Hindu-Arabic numerals, in computation. The Chinese use their own number symbols in formal writing but prefer to compute with the Hindu-Arabic numerals, which are much easier to use.

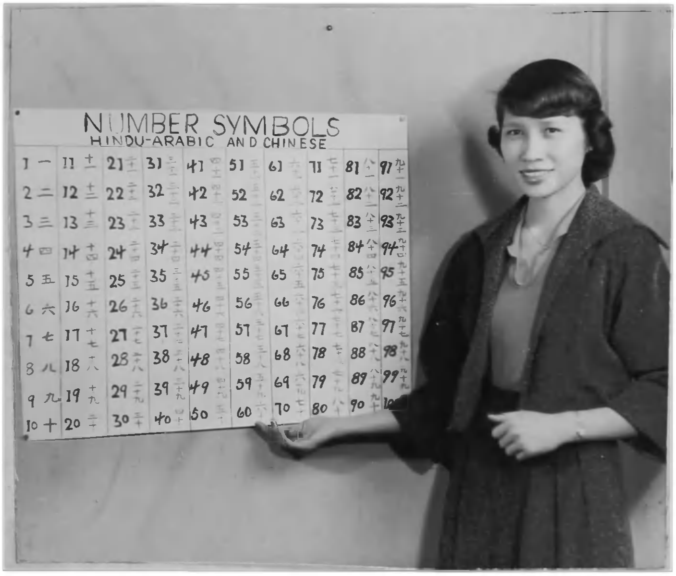

Plate II. Number Systems with a Base of Ten.

Miss Millie Jung, a native of China and a student at Southwestern Louisiana Institute, made a chart for the classes to show the similarity between the arrangement of the number symbols in the Chinese and the Hindu-Arabic number systems. Both number systems use ten as the base and rely on the principle of positional notation to extend the system. For example, Miss Jung indicated that in Chinese 60 is formed by writing the symbol for 6 above the symbol for 10.

Understanding the structure of numbers.

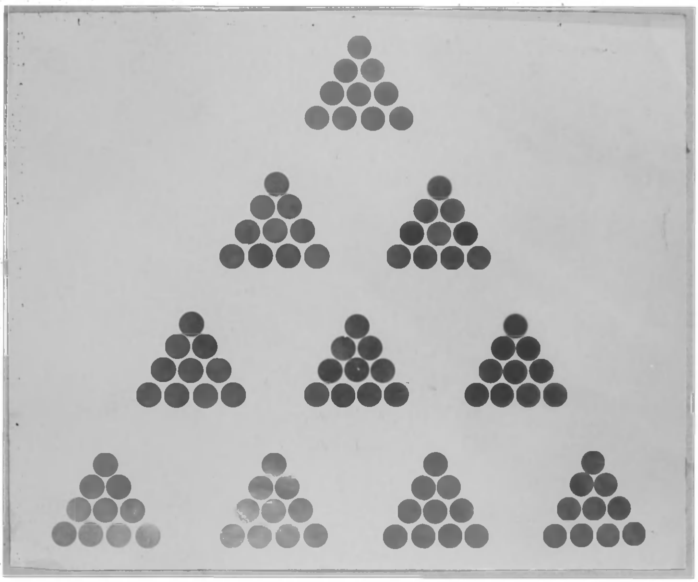

Plate III. A Configuration of 100.

The gummed one-inch dots were so arranged that the pupils sensed the unity of a group of ten made up of 10 ones and felt the unity of the larger group of one hundred made up of 10 tens. One of the pupils noticed that the order of the tens was the same as the order of the ones. This chart helped the children to see the decimal nature of the number system.

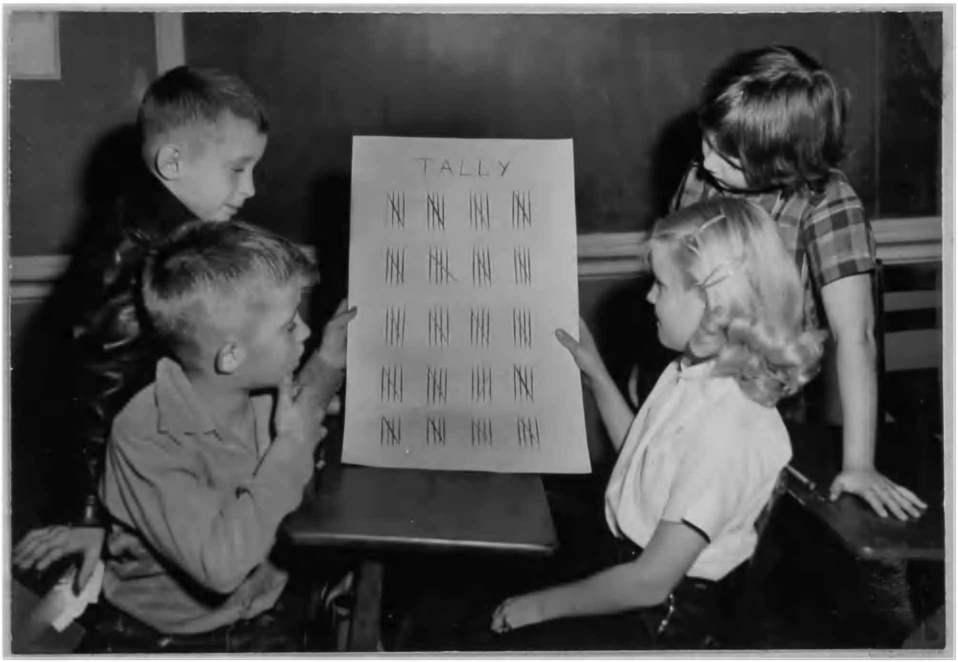

Plate IV. Studying Groups of Five.

Colored toothpicks glued to tagboard[6] were useful as tallies in studying groups of five. The pupils found twice as many groups of five in 100 as groups of ten.

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[6] Designed by Mrs. Shirley R. Lagneaux, third grade supervisor at F. M. Hamilton Training School, Lafayette, Louisiana.

Analyzing the story of ten.

Plate V. Writing the Facts about Ten.

The pupils used pictures in their textbooks and workbooks and drawings like the one illustrated here to study the corresponding addition and subtraction facts in the first decade.

Understanding the teen numbers.

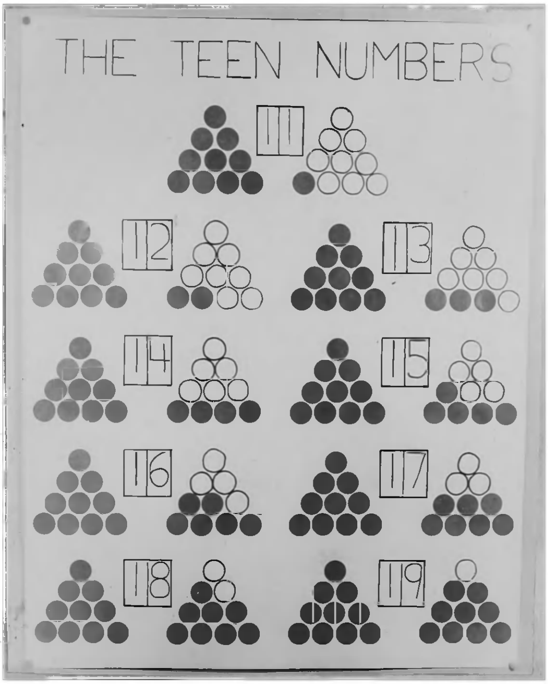

Plate VI. A "Moving Picture" of the Teen Numbers

The children found many interesting relationships from this chart. Kelly contrasted the structure of 11 and 19, 12 and 18, and the other related pairs. He noticed, for example, that 11 had as many ones toward 2 tens as 19 lacked being 2 tens.

<u>Finding answers in addition and subtraction</u>. A good understanding of the structure of the teen numbers paid great dividends in reconstructing some of the harder addition and subtraction facts which the children had trouble remembering. In the beginning the children found it difficult to add a number to nine, but later they saw that this was easy if they first added the same number to ten.

Plate VII. Using a Diagram to Find the Answer to 8 and 6.

Janet had forgotten 8 and 6. She found the answer by adding enough to the group of 8 to complete a group of 10.

The pupils were encouraged to think the rearrangements once they had sufficient experience with diagrams.

The answers to subtraction problems whose minuends were teen numbers were found in a similar way. If a pupil forgot 16 - 7, he was encouraged to first think of 16 as 10 and 6. Then the problem became simple:

$$\begin{array}{rl} \text{From} & 10 + 6 \\ \text{subtract} & \underline{7} \\ & 3 + 6\text{, or } 9 \end{array}$$

By subtracting 7 from the 10 and adding 3 to 6 he found the correct answer. This method worked well since the children had already mastered the addition and subtraction facts of the first decade.

The children were not limited to the use of any one method. There was always time to hear how different children knew or found the answer. Some children preferred to take 6 from 16 to get 10, and take 1 more from 10 for the final answer. They were shown that the methods were alike in that both depended on an understanding of the structure of the teen numbers. In the first method the pupil began by subtracting from the 10-group; in the second he began by taking away as many units as the minuend contained and taking the rest from the 10-group. The pupils saw that both methods were effective.

Once a fact was reconstructed the pupils were asked to practice thoughtfully saying the fact over and over.

Analyzing the teen numbers.

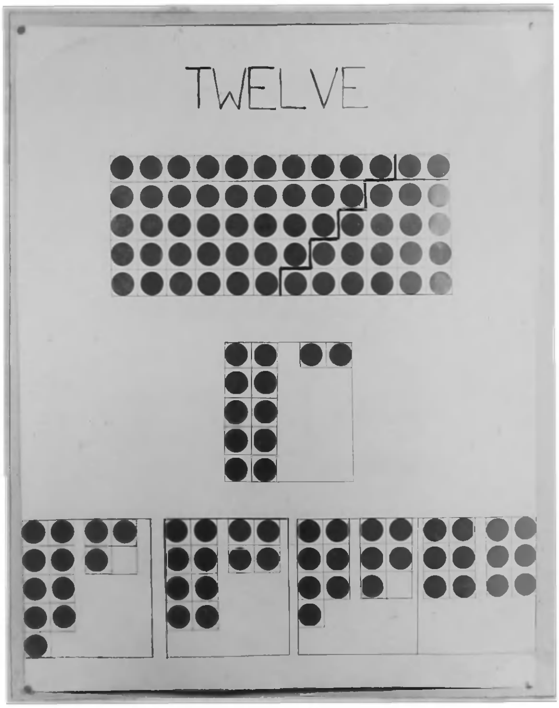

Plate VIII. Analysis of the 12-group.

In the arrangement at the top of the chart the sub-groups were combined to form the whole. This synthesis of the component parts allowed the pupils to see the parts in relation to each other as well as to the whole group. In the second arrangement a group of 10 and a group of 2 were formed, and in the analysis a "moving picture" was made by regrouping to get all the possible sub-groups. The pupils said that they liked to study the sub-groups in both ways. The first arrangement helped them with addition facts and the second helped with subtraction facts. Eleven was studied in a similar manner.

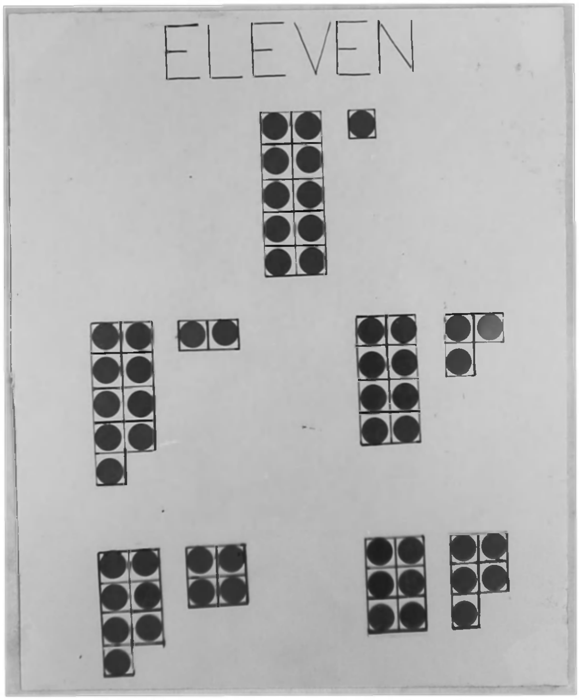

Plate IX. Analysis of the 11-group.

Studying odd and even numbers.

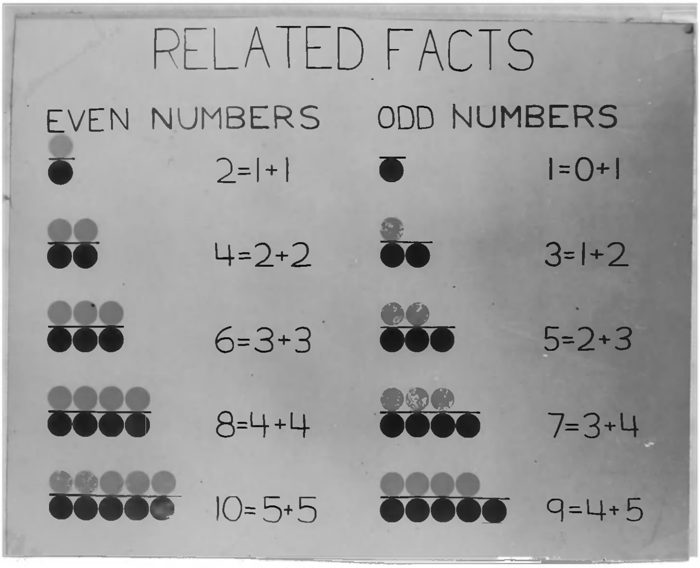

Plate X.

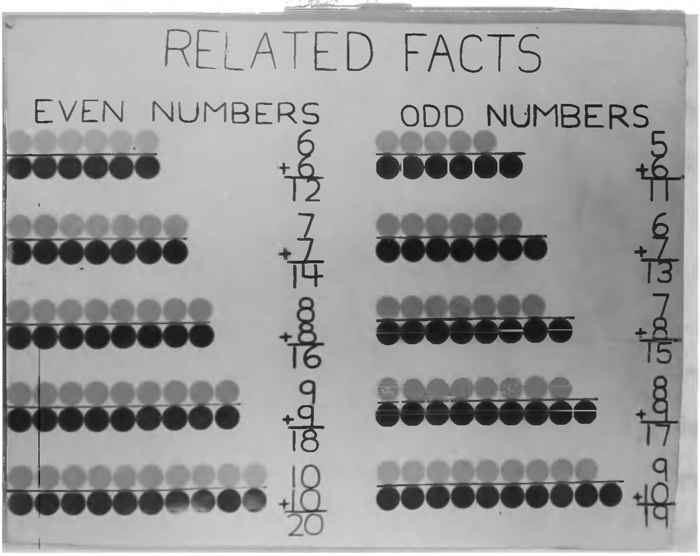

Plate XI.
Related Facts: Even and Odd Numbers.

In the charts illustrated on the preceding page the even numbers were separated into equal groups and the odd numbers into near-equal groups. The pupils could see the relationships between the easier addition facts, such as 8 + 8, and the more troublesome related facts, 7 + 8 and 8 + 9.

The nature of a typical learning experience may be shown by a discussion of a drill device the children sometimes used. They called it "a thinking game." The leader began by saying, "I'm thinking of an even number made up of the same two numbers, 7 and 7. What number am I thinking of?"

The child who answered became leader and would continue, "I'm thinking of an odd number made up of two numbers that follow each other, 6 and 7. What is the number?"

Most of the pupils found the doubles easy. They soon saw that for every double they knew, such as 7 + 7, they also knew two other related facts, 6 + 7 and 7 + 8. When a child had trouble remembering a double he learned to associate the fact with something in his daily experience. In one week there were 7 days, and in another week there were 7 days; so there were 14 days in all. In one carton there were 8 small packages of cereal. In two cartons there were 8 and 8, or 16. The pupils had attended school 9 months in the first grade and 9 months in the second grade. They had attended 9 and 9, or 18 months, of school in the first two grades.

Extending the addition and subtraction facts of the first decade to higher decades.

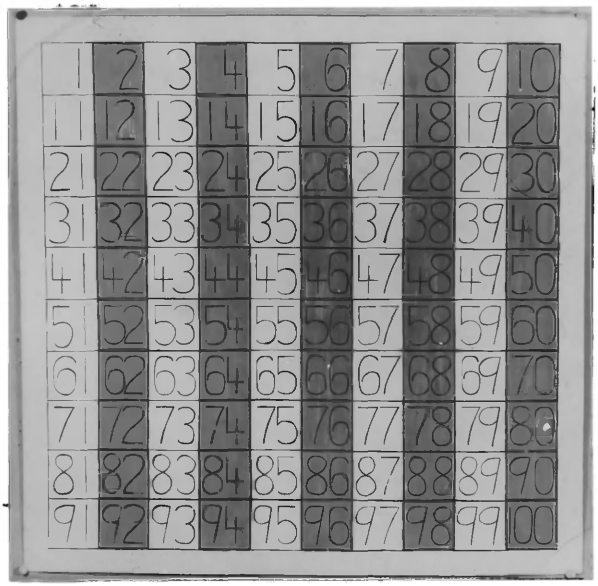

Plate XII. The Fact Finding Chart.

This chart with the even numbers in green squares and the odd numbers in white squares had many uses. Here are some principles that the pupils discovered:

1. Adding 2 to an even number gives the next larger even number. Subtracting 2 from an even number gives the next smaller even number.

2. Adding 2 to an odd number gives the next larger odd number. Subtracting 2 from an odd number gives the next smaller odd number.

3. Adding 10 to a number gives the number in the next decade which is in the same relative position. Subtracting 10 from a number gives the number in the next lower decade which is in the same relative position.

4. Adding 9 to a number gives one less than if 10 were added to the same number. Subtracting 9 from a number gives one more than if 10 had been subtracted from the same number.

5. If one knows an addition fact in the first decade, such as 3 + 4 = 7, then he knows nine other related facts:

13 + 4 = 17
23 + 4 = 27
33 + 4 = 37 . . .

The pupils were asked to write the facts in equation form to encourage a one-step thought process here. Nothing was said about adding by endings. A pupil located 3 with a pointer and moved on four units to reach 7. By placing the pointer in a vertical fashion on the third number in each decade, in a single move over four units, the seventh unit in each decade was reached.

The corresponding subtraction facts were discovered by reversing this procedure, counting back four units:

$$7 - 4 = 3$$
$$17 - 4 = 13$$
$$27 - 4 = 23 \ldots$$

6. Bridging in addition means a "spilling over" to the next higher decade. For example:

if $9 + 3 = 12$
then $19 + 3 = 22$
and $29 + 3 = 32 \ldots$

Bridging in subtraction means a "reaching down" into the next lower decade. For example:

if $12 - 8 = 4$
then $22 - 8 = 14$
and $32 - 8 = 24 \ldots$

<u>Understanding 3-place numbers.</u>

Plate XIII. The Ratio Meaning of Number.

Using the diagram made with a stencil chart illustrated on the preceding page, the pupils were able to visualize the enrollment of their school. Allen indicated that thirty-one pupils out of the 510 were in his class.

<u>Building readiness for multiplication and division by studying equal number-groups</u>. When the pupils had finished studying related addition and subtraction facts, they divided a factorable number into its equal groups. These activities were distributed throughout the semester's work as a part of the total study of the number-groups from 2 to 18.

Plate XIV. Multiplication Pairs.

Most multiplication facts, like most addition facts, go in pairs. Wayne found, by studying the charts illustrated above, two new facts about 18. He saw that 18 could be separated into three groups of 6 and six groups of 3. From these facts he saw multiplication as a short form of addition. Eighteen was 6 + 6 + 6 or 3 + 3 + 3 + 3 + 3 + 3.

Wayne could also find the related division facts. He answered these questions from the pictures:

"How many 6's in 18?"

"How many 3's in 18?"

Multiplication was seen as the combining of equal groups into a larger group. Division was seen as the separation of a larger group into equal groups. This chart illustrated the measurement idea of division, that is, the number of equal groups in a number.

Plate XV. Studying Groups by Counting.

The figures asked a question: Three 5's are how many? Counting by 5's gave the answer.

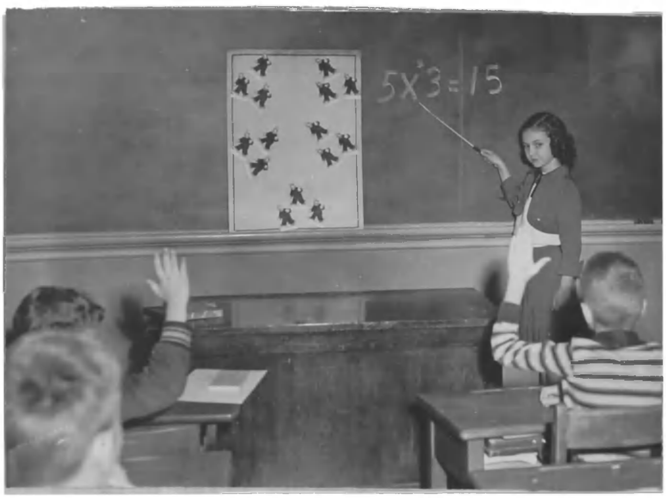

Plate XVI. Finding a New Multiplication Fact from a Related One.

Since three 5's were 15, then it followed that five 3's were also 15.

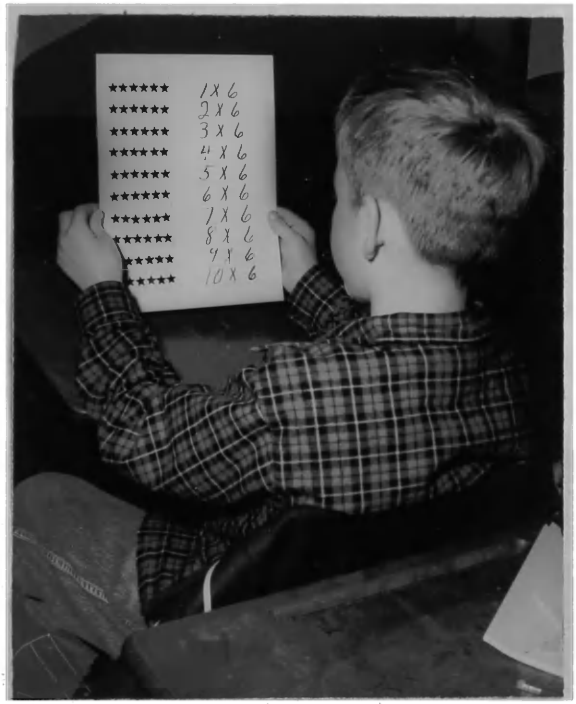

Plate XVII. Multiple Groups of Six.

Kirk knew that he brought 6¢ a day for school lunch, or 30¢ a week. In two weeks he brought just twice as much, or 60¢.

Talking about the pupils' lunch money provided an incidental learning experience which developed an understanding of the multiplications about the sixes which are not usually studied until grade four.

Studying multiplication pairs helped to emphasize the roles that the multiplier (the number of equal groups) and the multiplicand (the size of the group) played in multiplication. Although the teacher never used these terms the pupils were aware of their meaning.

Understanding the process of multiplication as a short way of adding equal groups and appreciating the role of the multiplicand and the multiplier enabled about half of the pupils in each group to help Josephine, a comic strip character, solve her problem: 3 X 23 is how many tens and how many ones?

Plate XVIII. Making the Transfer from Multiplying Ones to Multiplying Tens and Ones.

Morgan remarked to Tommy, "Josephine will have to come help me; I can't help Josephine."

Tommy showed Morgan two good ways to solve the problem: (1) by using 23 as an addend three times, and (2) by multiplying. Tommy reasoned, "Three 3's are 9, and three 20's are 60, so the answer must be 69."

Telling time. The experimenter supplemented the illustrations in the textbook and workbooks with drawings on the blackboard.

Plate XIX. Telling Time Associated with that Pleasant Experience, Recess.

Time was associated with the children's experiences. Each child, in both groups, worked up a chart called "A Time for Everything." He showed the time that he (1) got up, (2) ate breakfast, (3) came to school, (4) had arithmetic, (5) had recess, (6) ate lunch, (7) went home from school, (8) ate supper, and (9) went to bed. Each child shared his chart with the other members of the class.

Learning the value of coins. A money chart was prepared by Mrs. Rosemary Campbell, a student at Southwestern Louisiana Institute, and loaned to the investigator. This chart taught the coins, their value, and the number needed to make one dollar. The initial and spontaneous response of the children was to find out how much money was on the chart.

Plate XX. "Collecting silver dollars is my hobby," explains George.

Toward the close of the lesson George asked if he might have the silver dollar. He wanted to add it to his collection.

More about the yard. Lagniappe for both groups.

Plate XXI. A Yard of Candy.

It was nearly Christmas and a treat was in store. The yard of candy was a new way to see the 36 inches or 3 feet in a yard.

These selected learning activities are representative of the many experiences provided for both groups. The pupils had numerous opportunities to do creative thinking, to make their own discoveries.

SPECIAL MATERIALS FOR THE EXPERIMENTAL GROUP

In addition to the symbolic and pictorial materials already described for use with both groups, the investigator used manipulative materials in teaching the experimental group. These materials were introduced at any time there appeared to be a natural use for them in developing a new number concept -- or in clarifying and enlarging an old one.

The following original devices were used to build understanding of numbers:

1. A string of 100 spools. Fifty spools dyed yellow and fifty spools dyed blue were strung on heavy cord in groups of ten with the colors alternating. Loops at the ends of the cord made it possible to hang the string on two hooks at the front of the room. The pupils and their teacher could manipulate the spools in view of the entire class. Enough cord was left beyond that taken up by the spools to allow them to slide freely. Plate XXXI, page 79.

2. Strings of 100 beads. Corresponding to the string of spools, each pupil had a string of 100 beads which he could manipulate. Groups of ten pink and ten green beads were strung on waxed twine with the colors alternating. Plate XXXI, page 79.

3. Number racks. Nine separate racks were made to hold groups of spools representing the component parts of the number-groups from 2 to 10 inclusive. Each rack consisted of vertical dowel rods fitted into a rectangular base. The rods varied in number and height according to the size of the number-groups represented. The spools were glued together to represent the numbers 1 to 10. A total of 100 small spools was used. One set of spools representing the numbers from 1 to 10 was enameled red and the second set representing the numbers from 1 to 9 was enameled white. The two sets of spools and the nine racks made a most flexible device. Plate XXII, page 72.

4. Place holders. One hundred small wooden applicators dyed red and two small cans painted white and labeled "Tens" and "Ones" were used by each pupil in studying the principle of positional notation. The sticks were bundled into 10-groups with rubber bands. A large rubber band formed a 100-group of the ten 10-groups. Plate XXX, page 78.

5. Reversible green and orange counters. Fiber discs, one inch in diameter, were enameled green on one side and orange on the other. Each pupil had a small plastic bag containing 18 discs. These counters were used in group arrangements on the pupils' desks or in tucked charts. Plates XXVII and XXVIII, page 76.

6. Tucked charts. Tagboard, 8 inches by 11½ inches, was folded into five tucks 1½ inches deep which were divided by staples into 20 small pockets to fit the green and orange counters. A tagboard backing, 6 3/4 inches by 8 inches, was attached with scotch tape. Each pupil was provided with a tucked chart for use with his 18 counters. Plates XXVII and XXVIII, page 76.

7. Magnetic blocks and metal board. Small magnets were embedded in single cubes and in blocks[7] scored to show the number of units contained. These cubes and blocks represented the numbers from 1 to 10. A metal board, 14 inches by 18 inches, served as a convenient display for the blocks. Plate XXIX, page 77.

8. Peg board and rings. A wooden base, 16 inches by 4½ inches by 1½ inches, contained ten removable pegs, 3½ inches high and ½ inch in diameter, placed in a straight row one inch apart. Bone crochet rings 7/8 inch in diameter were used to form number-groups. Plate XXXIV, page 82.

9. Felt board and discs. A rectangle of all wool black felt, 36 inches by 18 inches, was thumbtacked to a

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[7] Unit blocks designed by Catherine Stern and sold by Houghton Mifflin Company, Dallas, Texas.

portable bulletin board and used as a base on which to manipulate all wool felt discs. The 18 discs were each made of two pieces of felt, 3 inches in diameter, glued together with rubber cement. The resulting discs were reversible, yellow on one side and rust on the other. The color, as well as the position of the discs on the felt board, helped to indicate the composition of the number-group.

The felt board was placed in the chalk tray at the front of the room so that all the pupils could see the number relations dramatized on it. Plates XXVII and XXVIII, page 76.

10. Flannel board. Gray flannel was mounted on a rectangle of cardboard, 22 inches by 28 inches. Pictures backed by fine sandpaper were used in forming and arranging number groups on the flannel board, which stood in the chalk tray at the front of the room.

11. The stair of numbers, 1 to 20. A metal board held all the magnetized unit blocks to form the series of numbers 1 to 20.[8] The teen numbers were studied in relation to the numbers in the first decade. Plate XXIII, page 73.

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[8] This is an adaptation of Catherine Stern's 20-case. Stern, *op. cit.*, p. 125.

12. <u>The teen rack</u>. This rack was built of wooden strips, 1 inch by 1 inch. It consisted of an upright 28 inches high to which were fastened five crosspieces 15 inches long, at intervals of 5 inches. In the top of each crosspiece was a groove ½ inch deep to hold carboard cut-outs of birds. There were ten red birds and ten yellow birds. The vertical support of the rack divided the cross-pieces into equal parts. On the left side two red birds were placed on each of the five branches to form a 10-group. The yellow birds were used on the right half in adding enough ones to form any of the teen numbers.

This aid was reserved for studying the teens as a 10-group and another smaller group. The children referred to the device as "the bird tree." Plates XXV and XXVI, page 75.

13. <u>Cartons of wooden eggs</u>. Egg cartons containing one dozen wooden eggs were used to show factors of 12. Plate XXXV, page 83.

SPECIAL METHODS WITH THE EXPERIMENTAL GROUP

<u>Manipulative materials, another medium of expression</u>. The additional use of manipulative materials gave the teacher and the pupils of the experimental group another way to represent numbers as groups and afforded a convenient way

to combine smaller groups and to separate larger groups. The pupils had some experience nearly every day in handling these materials. The length of time the aids were used depended on (1) the nature of the activity and (2) the maturity of the pupils using them.

Manipulative materials designed for teacher and pupils. The children not only watched the teacher manipulate the larger objects, but they manipulated their own smaller materials at their desks or gave demonstrations before the class with the larger materials which could be seen by all.

Manipulative materials used as thought models. The investigator was careful to keep the activity with these materials a thinking activity. The physical arrangements and re-arrangements were made: (1) to answer some question which required addition, subtraction, multiplication, or division, and (2) to give understanding to the numbers themselves. The materials were looked upon as a means to an end, as thought models, or as a springboard to independent work habits.

SELECTED LEARNING ACTIVITIES USING MANIPULATIVE MATERIALS WITH THE EXPERIMENTAL GROUP

<u>Analyzing the story of ten.</u>

Plate XXII. Discovering Facts about Ten.

This rack of spools made it possible to break the number 10 into its component parts. The children learned to tell and write the related addition and subtraction facts by families, thus:

9 + 1 = 10	10 - 9 = 1
1 + 9 = 10	10 - 1 = 9
8 + 2 = 10	10 - 8 = 2
2 + 8 = 10 . . .	10 - 2 = 8 . . .

These same spools were rearranged on other racks, one for each group from two to ten, as an aid to learning the rest of the related addition and subtraction facts in the first decade.

<u>Understanding the teen numbers</u>.

Plate XXIII. The Twenty Stair.

Each step in this stair of magnetic blocks on a metal board was one unit higher than the step below it. Each step representing a teen number was made by combining a 10-block and a smaller block. Jon pointed to 14, the number of days remaining before Christmas.

Plate XXIV. Comparing Groups with Peg Boards.[9]

Maureen showed the age of a younger sister with 6 pegs and the age of an older brother with 16 pegs, a group of 10 and a group of 6. She said, "It's easy to see that my brother is 10 years older than my little sister."

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[9] Aid designed and presented to the investigator by Mrs. Lagneaux.

Finding answers in addition and subtraction. The pupils found the more difficult addition and subtraction facts easier when they understood the structure of the teen numbers.

Plate XXV.

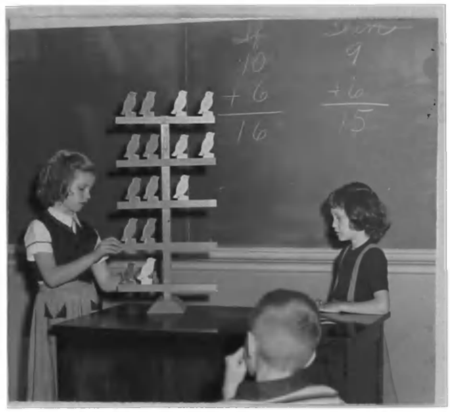

Plate XXVI.

Finding the Answer to 9 and 6.

Lynn Nell could not recall 9 and 6, but she said that she knew how to find the answer by using "the bird tree." First she placed red birds on the branches to form a group of 9. Then she used yellow birds to form a group of 6. She thought, "Nine and 6 will be 10 and some more." So she completed the 10-group with one yellow bird, regrouping 9 and 6 as 10 and 5 to get 15.

Audrey Anne watched Lynn Nell find the answer, then said that she got the answer another way. She thought, "Ten and 6 is 16, so 9 and 6 is one less, or 15."

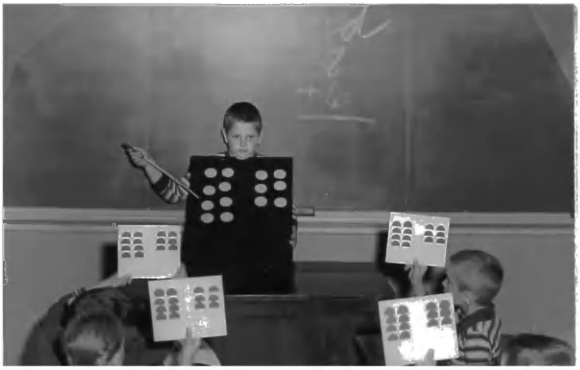

Plate XXVII. How many are 8 and 6?

Mike used a felt board with felt discs to represent the groups of 8 and 6. The other pupils used tucked charts and formed the groups with green discs. Mike asked, "Will the answer be more than 10?"

"Yes," the pupils replied.

"Then regroup your discs to find the answer," suggested Mike.

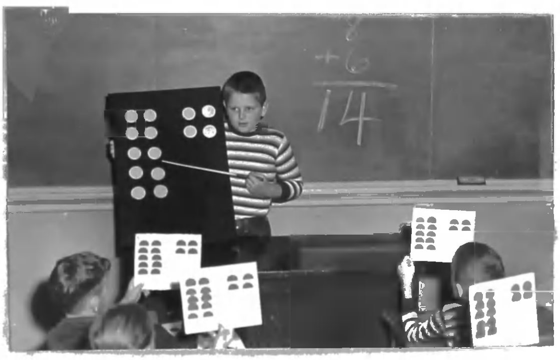

Plate XXVIII. Rearranging the Discs to Find the Answer.

Analyzing the teen numbers.

Plate XXIX. Studying the
Component Parts of 12.

The magnetic blocks were used to tell the story of 12. Robert started it with a 10-block and a 2-block. Dianne added a 9-block and a 3-block. The children took turns building the other combinations. Later they separated the groups of blocks to tell the related subtraction facts.

Understanding 2-place numbers.

Plate XXX. Showing the Structure of 31.

Each pupil used two small cans as place holders, one labeled "Tens" and the other labeled "Ones." With 100 sticks, divided into bundles of 10, the structure of any 2-place number could be quickly represented. Lynn Nell and the other pupils represented 31 with 3 bundles of ten in the 10's place and a single stick in the units' place.

These sticks and cans were used in the addition and subtraction of tens and ones and to demonstrate the principle of bridging. In addition and multiplication the bridging was seen as the regrouping of 10 units of a smaller denomination into 1 unit of the next larger denomination.

In subtraction and division the bridging was seen as changing 1 unit of a larger denomination for 10 units of the next smaller denomination.

Plate XXXI. Making a Picture of 43.

Using the string of 100 spools Marty Ann pictured 43 as 4 groups of ten and 3 ones. The other pupils carried out the activity with their strings of 100 beads.

Understanding 3-place numbers.

Plate XXXII. Building 510.

A set of blocks to represent ones, tens and hundreds[10] was used to give meaning to numbers between 100 and 1000. Don and Berenda used five 100-blocks and one 10-block to show the enrollment at Myrtle Place School.

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10 These blocks were made for the investigator from a diagram of Stern's. Catherine Stern, Children Discover Arithmetic (New York: Harper and Brothers, 1949), p. 267.

Plate XXXIII. Using a Chinese Abacus to Show 510.

Miss Jung represented 510 in another way, showing the use of the abacus. She explained to the children that the order of the rods determined the denomination. The bottom rod marked the ones' place, the one above, the tens' place, etc. The beads on the left of the dividing bar represented one unit of the denomination, and the beads on the right stood for 5 units of the denomination. Miss Jung moved over one bead on the third rod, representing 5 of the hundreds, and one bead on the left half of the second rod, representing 1 ten.

Building readiness for multiplication and division by studying equal number-groups.

Plate XXXIV. Using a Peg Board to Show Multiple Groups of Six.

The ten pegs on the board each contained six rings to represent the amount of money brought daily for school lunches during a two-week period.

Plate XXXV. Multiplication Pairs Found in Mother's Kitchen.

Martin had a carton of a dozen eggs divided into two rows. He could see the multiplication pairs, two 6's and six 2's. David had another carton with a different arrangement, three 4's and four 3's.

From these cartons the following multiplication and division facts were found:

$$\begin{array}{r}6\\ \times\ 2\\ \hline\end{array} \qquad \begin{array}{r}2\\ \times\ 6\\ \hline\end{array} \qquad 6\overline{)12} \qquad 2\overline{)12}$$

$$\begin{array}{r}4\\ \times\ 3\\ \hline\end{array} \qquad \begin{array}{r}3\\ \times\ 4\\ \hline\end{array} \qquad 4\overline{)12} \qquad 3\overline{)12}$$

Telling time. Each pupil made a clock for his own use. In addition the big Judy Clock[11] was used for

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[11] Supplied by The Judy Company, 310 North Second Street, Minneapolis 1, Minn.

demonstration purposes before the group. Reference was made to a large electric wall clock with a second hand. Time was studied on the hour, half hour, quarter hour, and to the minute. The children learned to state and record time in a variety of ways.

Plate XXXVI. What Time Is It?

"It's time for arithmetic," said Sandra. "What time is it?" She received the following responses:

"It's fifteen minutes to nine."

"It's eight forty-five."

"It's a quarter to nine."

<u>Introducing the partition idea of division.</u> The partition idea of division was approached through sharing. On the Stanford Achievement Test, Form K, not one pupil had solved the problem, 2/186. The pupils had shared ones and tens, but they had no experience in sharing hundreds, tens, and ones.

The teacher posed the problem this way: "Suppose you had $186 and wished to share it equally between two of your friends. How would you do it?"

Plate XXXVII. Representing 186 as
1 Hundred, 8 Tens, and 6 Ones.

Tommy said he believed he knew how to share the money but if he <u>really</u> had that much he might not want to give it to two of his friends.

Tommy saw that he could not divide the 100-block. So he decided to exchange it for 10 tens which he could divide.

Plate XXXVIII. Using 186 as 18 Tens and 6 Ones.

Tommy took half of the 18 tens and gave each boy his share, 9 tens. He then took half of the 6 ones and gave each boy 3. Tommy finally told his friends, "I'm sorry this isn't <u>real</u> money, but I gave each of you the same amount, didn't I?"

The activities described above are typical of the additional experiences provided only for the experimental group. The use of manipulative materials gave the pupils another way to represent numbers as groups and to dramatize the processes as groups in action.

CHAPTER V

PRESENTATION OF THE RESULTS

In December, after three and one-half months of instruction, the groups were tested again for arithmetic achievement. The original objective test was repeated; and an alternate form of the Stanford Achievement Test, Primary Battery, this time Form K, was administered.

In order to determine the relative effectiveness of the two methods used with the control and experimental groups, both intra-group and inter-group comparisons were made. Intra-group comparisons were made to find whether each group had shown growth that was significant for the particular method used with that group. Inter-group comparisons were made to find whether the manipulative materials used additionally with the experimental group had produced any significant difference in growth for this group over that for the control group, which had not had access to the manipulative materials.

The significance of the difference between the means on the testings was the method used to make the comparisons. Since the groups were small a better estimate of the standard deviation for the intra-group comparisons was obtained by pooling the sums of the squares of the deviations from the means on the testings in September and in December of each group of children. Likewise, for the inter-group comparisons a better estimate was obtained by pooling the sums of the squares of the deviations from the means on the final testings of the two groups in December. The inter-group comparisons of the upper thirds, middle thirds, and lower thirds of the two groups were made in a similar manner. (See formulas, page 21.)

INTRA-GROUP COMPARISONS

The growth in arithmetic achievement of each group, determined by a comparison of the grade placements on the Stanford Achievement Tests in September and in December, is shown in Table VII.

TABLE VII

GROWTH IN ACHIEVEMENT OF EACH GROUP ON THE STANFORD TESTS*

Group	Number Sept. and Dec.	Mean grade placement Sept.	Mean grade placement Dec.	Mean difference	Standard deviation or s	Standard error or s_D	Critical ratio or t
Control	54[x]	2.7	3.4	.7	.59	.11	6.36
Experimental	54[x]	2.7	3.4	.7	.59	.11	6.36

* Based on information in Tables XIII and XV, Appendix A.

x Pooling 27 cases in September with 27 cases in December.

The critical ratios in Table VII above indicate that there was significant growth in arithmetic achievement within both groups. With 50 degrees of freedom a critical ratio of 2.40 is significant at the .01 level, and the critical ratios found in the above table are greatly above this.

The mean grade placement for each group was 3.4 in December as compared to 2.7 in September. In three and one-half months the average gain for each group was seven months, that is, two months of growth for each month of instruction.

Table VIII shows the growth in achievement on the original objective test.

TABLE VIII

GROWTH IN ACHIEVEMENT OF EACH GROUP ON THE ORIGINAL OBJECTIVE TEST*

Group	Number Sept. and Dec.	Mean scores		Mean difference	Standard deviation or s	Standard error or s_D	Critical ratio or t
		Sept.	Dec.				
Control	52[x]	97	117	20	22.9	6.41	3.20
Experimental	52[x]	100	120	20	20.9	5.85	3.40

* Based on information in Tables XIV and XVI, Appendix A.

x Pooling cases in September and December. Groups reduced by absence of one pupil for a testing.

The mean score for the control group on the original objective test in September was 97 as compared to 117 in December. The mean score for the experimental group was 100 in September and 120 in December. Both groups thus showed a net gain of 20 points on the final test. The critical ratios were 3.20 for the control group and 3.40 for the experimental group, far in excess of the 2.40 usually regarded as indicating a significant difference at the .01 level.

These highly significant critical ratios obtained for both the Stanford Test and the original objective test indicate that there was significant growth in achievement within each group during the period of instruction.

Growth in achievement was also shown by the reduction of underachievers in both groups, when each child's achievement was compared with what might be expected of him in relation to his mental ability. The mental age grade placements made by the pupils on the California Test of Mental Maturity were converted to intelligence quotients in order to compare intelligence and achievement. A table[1] of norms in the Manual of the California Test of Mental Maturity shows what variation in arithmetic achievement may be expected in grade three for groups with varying median

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[1] Elizabeth T. Sullivan, Willis W. Clark, and Ernest W. Tiegs, Manual, California Test of Mental Maturity (Los Angeles: California Test Bureau, 1951), p. 16.

intelligence quotients. Identifying individual I.Q.'s with corresponding median I.Q.'s on the table, it was possible to compare each child's achievement and his mental ability. The norm was chosen as 3.0 in September and 3.4 in December. Pupils with I.Q.'s of 100 were expected to achieve the normal grade placements; whereas, those with I.Q.'s above 100 were expected to achieve grade placements above the norms for each testing, and those with I.Q.'s below 100 were not expected to achieve up to the grade norms.

Table IX shows the reduction in the number of under-achievers during the period of instruction.

TABLE IX

ACHIEVEMENT IN RELATION TO ABILITY, AS REVEALED BY THE STANFORD ACHIEVEMENT TESTS AND THE CALIFORNIA TEST OF MENTAL MATURITY

Group	Number of pupils whose achievement did not equal ability		Number of pupils whose achievement equaled or exceeded ability	
	Sept.	Dec.	Sept.	Dec.
Control	20	9	7	18
Experimental	18	9	9	18
Totals	38	18	16	36

The investigator used the table of I.Q.'s and the expected grade placements in the manual with caution for two reasons: (1) the I.Q.'s of children from low social-status levels do not always give a true picture of their mental capacity, and (2) children from disturbed homes often lack the proper motivation to do their best work when they take tests. In order to interpret the I.Q.'s of any group properly, the relation between intelligence and cultural differences must be understood.[2]

For example, two pupils from families of unfavorable socio-economic backgrounds may be considered. In the control group there was a pupil with an I.Q. of 83, whose rank was 26th, next to the bottom for his group, and whose mother was in a mental institution. In September his grade placement on the Stanford Achievement Test was 1.9. In December his grade placement was 3.0. This achievement was phenomenal; it represented 11 months growth during 3½ months of instruction. This growth far exceeded the expected achievement based on his I.Q.

In the experimental group there was a pupil with an I.Q. of 88, whose rank was 25th, third from the bottom of his class, and whose mother was in a tuberculosis sanitorium.

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[2] Allison Davis, *et al.*, *Intelligence and Cultural Differences* (Chicago: The University of Chicago Press, 1951), pp. 1-47.

In September his grade placement on the Stanford Test was 2.2 and in December it was 3.2. His gain represented one whole year in arithmetic achievement. His growth, like the other pupil's, far exceeded what might have been expected according to his I.Q.

The unusual progress of these pupils may be partially explained in two ways: (1) by encouragement from the teacher, and (2) by reasonable success of the pupils. Encouragement and success were two important motivating factors in reducing the total number of underachievers in arithmetic in both groups.

All intra-group comparisons point up the same fact: that each method used produced significant growth *within* its group.

INTER-GROUP COMPARISONS

Since both groups of pupils were equated as nearly as possible in arithmetic achievement in September, then any significant difference in growth between the groups on the final testing would indicate whether one method of instruction was more effective than the other.

In Table X the results of the final testings are compared for the control group and the experimental group.

TABLE X

COMPARISON OF GROWTH BETWEEN CONTROL AND EXPERIMENTAL GROUPS ON ARITHMETIC ACHIEVEMENT IN DECEMBER*

Test	Number of cases Cont.	Exp.	Mean grade placements or scores Cont.	Exp.	Mean difference	Standard deviation or s	Standard error or s_D	Critical ratio or t
Stanford	27	27	3.4	3.4	0			Zero
Original	26[x]	26[x]	117	120	3	19.70	5.52	.54

* Based on information in Tables XIII to XVI, Appendix A.

x One pupil missed a testing.

This table shows that there was no difference between the mean achievement of the two groups on the Stanford Test on the final testing, and the difference between the means on the original test was only a slight one. The critical ratios of zero and .54 indicate that the difference in growth between the groups was not significant.

Might the use of manipulative materials have been advantageous, however, to a particular group, the above-average, the average, or the below-average pupils? Table XI compares the growth of the upper thirds, the middle thirds, and the lower thirds of the experimental and control groups, and reveals no significant difference.

TABLE XI

COMPARISON OF GROWTH IN ARITHMETIC ACHIEVEMENT BETWEEN SUB-GROUPS OF THE EXPERIMENTAL AND CONTROL GROUPS, DECEMBER*

Sub-group	Number Exp.	Number Con.	Mean Exp.	Mean Con.	Mean difference	Standard deviation or s	Standard error or s_D	Critical ratio or t
Stanford Achievement Test								
Upper thirds	9	9	3.8	3.7	.1	.27	.13	.77
Middle thirds	9	9	3.3	3.4	.1	.42	.20	.50
Lower thirds	9	9	3.2	3.0	.2	.28	.13	1.54
Original objective test								
Upper thirds	9	9	135	132	3	6.90	3.24	.93
Middle thirds	9	9	116	124	8	12.60	5.90	1.36
Lower thirds	9	9	107	97	10	20.50	9.64	1.04

* Based on information in Tables XIII to XVI, Appendix A.

It will be recalled that with the 18 cases resulting from pooling the small samples there are 16 degrees of freedom, and to be significant at the .01 level the critical ratio must be 2.58. The critical ratios in Table XI on the preceding page are far below 2.58, indicating no significant differences in growth between the groups.

The evidence of both inter-group comparisons and intra-group comparisons of growth indicates clearly that:
(1) the growth within each group was significant, and
(2) neither the difference in growth between the groups nor the differences in growth between the sub-groups was significant. In other words, both methods proved equally effective with the group in which they were used. The additional use of manipulative materials with the experimental group caused no significant difference in the growth of that group over the growth of the control group, with which only the symbolic and pictorial materials were used. Nor was there any significant difference between the growth of the upper thirds, the middle thirds, or the lower thirds of the two groups.

CHAPTER VI

SUMMARY AND CONCLUSIONS

SUMMARY

The investigator equated two groups of third grade pupils at Myrtle Place School, in Lafayette, Louisiana, in order to determine the effectiveness of the use of manipulative materials in teaching arithmetic.

Both groups of children were taught by the investigator for daily periods of thirty-five minutes each during the fall semester of 1953-54. The same textbook and workbooks were used by both groups. The investigator designed supplementary symbolic and pictorial materials, as shown in the body of the thesis, and used these materials alike with both groups. These materials used alike by the same teacher with the same overall objectives constituted the control factor in the study. The experimental group had the additional and exclusive use of the manipulative materials. The additional use of these materials with this group provided the variable factor in the study.

The laboratory method, which stressed pupil discovery, was used with both groups. Arithmetic was taught as an orderly, systematic way of studying numbers as groups.

After three and one-half months of instruction the children were tested again to determine their status in arithmetic achievement. Intra-group comparisons were made to measure the effectiveness of each particular method with each group. Inter-group comparisons were made to measure the difference in achievement between the two groups.

CONCLUSIONS

The intra-group comparisons showed that each method produced significant growth within its own group. The inter-group comparisons revealed no significant difference between the growth of the control group and the experimental group; nor were there any significant differences in growth between the upper thirds, the middle thirds, and the lower thirds of the two groups.

It may be concluded from the study:

First, that within the limits of grade objectives to which this study was confined, carefully selected pictorial and symbolic materials provided adequate instruction.

And second, that the additional use of manipulative materials with the experimental group caused no significant gain in growth of that group over the control group.

It should be pointed out again that this study was limited in scope to the arithmetic objectives (i.e., concepts and skills) usually found in the course of study for the first half of the third grade, and the tests for measuring achievement were developed within these limited grade objectives and from the course of study materials laid out for this grade. However, the study indicates conclusively that the objectives of this third grade, as laid out in the school courses of study, can be met adequately by the use of textbook materials, supplemented richly by other pictorial and symbolic materials such as those shown in the body of this study. Stated precisely, this study showed no significant advantage for the group that had access to the manipulative materials, as measured by their learning when confined within the limits of the third grade course of study, first half, and especially when the tests were also restricted within these limits.

This finding does not preclude the possibility, however, that if the study could be repeated with two groups in such a way that their learning were not restricted within rather narrow course of study limits and, particularly, if the tests were not also thus restricted, there might be found an advantage in the use of the manipulative materials. This merely indicates the need in this area for more study in different directions.

Since this is a pioneer study in testing the effectiveness of manipulative materials in teaching arithmetic, the investigator suggests that the experiment be repeated by other teachers on different grade levels and with broader testing objectives than those usually developed for and standardized within specific grade levels.

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APPENDIX A

REFERENCE TABLES
TABLE XII

CHRONOLOGICAL AGES AND MENTAL AGES*
OF PUPILS IN SEPTEMBER

Rank[x]	Control group Age	Control group Grade placement	Control group I.Q.	Experimental group Age	Experimental group Grade placement	Experimental group I.Q.
1	7-10	3.4	111	8- 5	4.7	118
2	8- 5	4.8	119	8- 4	4.9	122
3	8- 8	3.4	105	7- 8	5.2	136
4	8- 8	4.5	113	8- 3	3.3	104
5	7-10	3.9	113	8- 4	2.8	96
6	7-11	2.8	101	8- 7	4.1	109
7	8- 2	3.0	101	7- 9	2.6	101
8	8- 3	3.3	104	8- 5	3.5	104
9	7- 8	3.5	114	8- 3	3.7	108
10	8- 5	2.8	95	8- 7	4.6	115
11	8- 5	2.2	87	8- 7	1.8	82
12	8- 0	3.7	113	7-10	3.1	106
13	8- 0	2.6	98	8- 1	3.4	107
14	7- 8	3.3	112	9- 3	2.6	85
15	8- 9	2.6	87	8- 7	2.4	88
16	8- 7	2.8	93	9- 0	4.8	111
17	8- 0	2.7	99	8- 7	4.0	108
18	8- 7	1.9	83	8- 0	2.2	92
19	8- 0	2.3	94	8- 2	1.6	83
20	7- 6	3.0	110	8- 5	2.9	97
21	10- 0	3.7	89	8- 2	3.2	103
22	8- 0	2.6	97	8- 8	2.6	90
23	9-10	4.7	101	8- 3	1.8	85
24	8- 8	2.7	91	8- 5	4.0	110
25	8- 3	0.7	70	8- 4	2.2	88
26	7-11	1.5	88	8- 2	1.4	80
27	7-10	5.1	128	8- 1	1.6	84

* Intelligence quotients and grade placements as determined by the California Test of Mental Maturity.

[x] According to rank on Stanford Achievement Test in September.

TABLE XIII

GRADE PLACEMENTS OF THE CONTROL GROUP
ON THE STANFORD ACHIEVEMENT TESTS
IN ARITHMETIC

Rank*	Reasoning Sept.	Reasoning Dec.	Computation Sept.	Computation Dec.	Total Sept.	Total Dec.	Grade gained
1	3.4	4.2	3.5	4.1	3.5	4.2	.7
2	3.8	4.5	3.0	3.9	3.4	4.2	.8
3	3.4	3.9	3.0	3.3	3.2	3.6	.4
4	3.4	3.9	2.8	3.3	3.1	3.6	.5
5	3.2	3.8	2.8	3.1	3.0	3.5	.5
6	3.1	3.8	2.9	3.4	3.0	3.6	.6
7	2.9	3.4	2.8	3.3	2.9	3.4	.5
8	2.9	3.8	2.8	3.8	2.9	3.8	.9
9	2.5	3.4	3.1	3.3	2.8	3.4	.6
10	2.7	3.6	2.9	3.0	2.8	3.3	.5
11	2.5	4.5	2.9	3.5	2.7	4.0	1.3
12	2.5	4.5	2.9	3.4	2.7	4.0	1.3
13	2.3	3.6	2.9	3.4	2.6	3.5	.9
14	2.3	2.8	2.8	2.9	2.6	2.9	.3
15	2.1	2.8	3.0	3.0	2.6	2.9	.3
16	2.5	3.8	2.7	3.0	2.6	3.4	.8
17	2.5	3.4	2.7	2.9	2.6	3.2	.6
18	2.3	3.2	2.9	3.1	2.6	3.2	.6
19	2.1	3.0	2.7	2.6	2.4	2.8	.4
20	1.8	3.8	2.9	3.5	2.4	3.7	1.3
21	2.3	3.1	2.4	2.6	2.4	2.9	.5
22	2.1	3.6	2.6	2.9	2.4	3.3	.9
23	2.1	3.0	2.7	2.9	2.4	3.0	.6
24	2.3	3.1	2.0	2.7	2.2	2.9	.7
25	1.6	2.6	2.6	2.6	2.1	2.6	.5
26	1.5	3.2	2.3	2.8	1.9	3.0	1.1
27	1.6	3.0	2.0	2.6	1.8	2.8	1.0

* According to rank on Stanford Achievement Test in September.

TABLE XIV

SCORES OF THE CONTROL GROUP ON THE ORIGINAL OBJECTIVE TEST IN ARITHMETIC ACHIEVEMENT

Rank*	Understandings Sept.	Understandings Dec.	Problems Sept.	Problems Dec.	Computations Sept.	Computations Dec.	Total Sept.	Total Dec.
1	73	68	15	18	45	44	133	130
2	70	79	16	18	44	45	130	142
3	68	82	10	omitted	43	39	121	121
4	56	82	13	15	36	40	105	137
5	58	74	inc.	12	--	42	inc.	128
6	54	76	13	18	43	43	110	137
7	49	65	11	13	31	41	91	119
8	61	80	12	16	43	44	116	140
9	68	76	8	14	43	44	119	134
10	46	75	1	10	29	43	76	128
11	49	75	9	16	35	41	93	132
12	67	78	12	17	38	44	117	139
13	62	74	11	8	43	41	116	123
14	58	75	11	13	44	45	113	133
15	53	63	9	10	42	43	104	116
16	58	60	15	9	39	41	112	110
17	46	67	9	9	34	33	89	109
18	53	71	11	14	36	45	100	130
19	34	38	7	7	33	37	74	82
20	55	76	10	16	40	44	105	136
21	31	49	0	7	20	26	51	82
22	45	59	11	11	34	33	90	103
23	51	73	7	17	42	43	100	133
24	34	55	4	6	38	41	76	102
25	31	42	1	9	21	30	53	81
26	32	57	6	omitted	22	34	60	91
27	34	49	5	omitted	19	14	58	63

* According to rank on Stanford Achievement Test in September.

TABLE XV

GRADE PLACEMENTS OF THE EXPERIMENTAL GROUP ON THE STANFORD ACHIEVEMENT TESTS IN ARITHMETIC

Rank*	Reasoning		Computation		Total		
	Sept.	Dec.	Sept.	Dec.	Sept.	Dec.	Gain
1	4.0	4.8	3.1	3.3	3.6	4.1	.5
2	3.8	4.2	3.0	3.8	3.4	4.0	.6
3	3.2	4.2	3.2	3.6	3.2	3.9	.7
4	3.2	3.8	2.9	3.5	3.1	3.7	.6
5	3.1	4.5	3.1	3.5	3.1	4.0	.9
6	3.1	3.9	3.1	4.1	3.1	4.0	.9
7	3.2	3.6	2.8	3.5	3.0	3.6	.6
8	3.2	3.8	2.7	3.2	3.0	3.5	.5
9	2.7	3.4	3.3	3.7	3.0	3.6	.6
10	2.9	4.9	2.9	3.3	2.9	4.1	1.2
11	2.7	3.2	3.0	3.1	2.9	3.2	.3
12	2.7	4.2	2.9	3.3	2.8	3.8	1.0
13	2.5	2.8	2.8	2.8	2.7	2.8	.1
14	2.5	3.2	2.8	3.1	2.7	3.2	.5
15	2.5	3.6	2.8	3.2	2.7	3.4	.7
16	2.3	3.1	2.8	3.1	2.6	3.1	.5
17	2.5	3.1	2.7	2.8	2.6	3.0	.4
18	2.5	3.1	2.7	2.6	2.6	2.9	.3
19	2.7	3.2	2.4	3.0	2.6	3.1	.5
20	2.1	3.6	2.8	3.2	2.5	3.4	.9
21	2.7	3.6	2.2	3.3	2.5	3.5	1.0
22	1.9	3.1	2.7	2.5	2.4	2.9	.5
23	2.1	3.9	2.6	3.0	2.4	3.5	1.1
24	2.3	3.8	2.5	2.8	2.4	3.3	.9
25	2.3	3.4	2.0	3.0	2.2	3.2	1.0
26	2.1	3.1	1.9	2.8	2.0	3.0	1.0
27	1.9	3.6	1.8	2.3	1.9	3.0	1.1

* According to rank on Stanford Achievement Test in September.

TABLE XVI

SCORES OF THE EXPERIMENTAL GROUP ON THE ORIGINAL OBJECTIVE TEST IN ARITHMETIC ACHIEVEMENT

	Understandings		Problems		Computations		Total	
Rank*	Sept.	Dec.	Sept.	Dec.	Sept.	Dec.	Sept.	Dec.
1	76	79	16	16	41	45	133	140
2	75	80	15	16	32	44	122	140
3	67	78	11	15	41	32	119	125
4	71	80	15	15	43	44	129	139
5	66	77	15	17	45	44	126	138
6	68	78	18	17	44	43	130	138
7	65	72	15	13	38	44	118	129
8	64	75	10	15	38	42	112	132
9	71	78	13	16	43	43	127	137
10	62	76	15	14	34	44	111	134
11	66	68	7	11	38	35	111	114
12	69	77	9	14	41	41	119	132
13	37	59	4	8	29	32	70	99
14	30	56	6	12	44	37	80	105
15	63	73	17	17	33	40	113	130
16	56	71	7	13	33	41	96	125
17	62	61	inc.	4	inc.	36	inc.	101
18	25	55	0	10	39	40	64	105
19	31	59	8	13	37	44	76	116
20	32	70	10	12	34	43	76	125
21	51	60	11	14	42	38	104	112
22	30	65	11	15	37	42	78	122
23	46	56	0	8	36	39	91	103
24	35	48	8	12	32	38	75	98
25	48	66	8	8	45	41	101	115
26	29	50	6	9	28	38	63	97
27	30	52	7	7	18	16	55	75

* According to rank on Stanford Achievement Test in September.

APPENDIX B

ARITHMETIC ACHIEVEMENT TEST

DIRECTIONS FOR ADMINISTERING

<u>Time</u>. Twenty-five minutes are allowed for Test I, with a five-minute rest period following it. Twenty minutes are allowed for Tests II and III together. The total working time allowed the pupils is forty-five minutes.

<u>Directions for scoring</u>. There are 147 items on the test. One point should be allowed for each exercise answered correctly. A few exercises have more than one response. No partial credit is allowed.

TEST I

CONCEPTS AND BACKGROUND

After the booklets have been distributed, say to the pupils: "Now open your booklet to Test I, Concepts and Background, on page 1."

<u>Specific Directions</u>:

Exercise 1.
"How many stars do you see? Draw a ring around the figure that tells how many."

Exercise 2.
"How many squares do you see? Draw a ring around the figure that tells how many."

Exercise 3.
"How many circles do you see? Draw a ring around the figure that tells how many."

Exercise 4.

"Count the boxes and write the figure in the blank to show how many there are."

Exercise 5.

"Count the trees and write the figure in the blank to show how many there are."

Exercise 6.

"Count the balls and write the figure in the blank to show how many there are."

Exercises 7-9.

"Here are ten children lined up to take a ride on the ferris wheel. Write their names or places in the blanks.

"_____ is first for the ferris wheel.

"Jack is _____ in line.

"_____ is eighth in line."

Exercises 10-12.

"After the ride each child found his same place in line, turned around and went across the fair grounds to buy some cotton candy at a stand. Write in the blanks their names or places in line.

"Lou is _____ to get the cotton candy.

"_____ is fourth.

"_____ is last."

Exercises 13-15.

"The third grade room could have one school party during the year. The class voted to see when they would have their party. The marks show the way they voted.

"How many children voted for the Halloween Party? Write the number on the blank beside the tallies. (Pause.)

"How many children wanted the Christmas Party? Write the number on the blank. (Pause.)

"How many voted for the Easter Egg Hunt? Write the number on the blank."

Exercises 16-23.

"Mary told the class that she had 9 little chickens at home. She said, 'Some of the chickens are black and some are yellow'. The class had fun guessing the number of each. Fill in the blanks to show all the guesses that were made."

Exercises 24-32.

"When Mary fed the chickens they ran to her one at a time. Finish the story to show how many were left with the mother hen each time."

Exercise 33.

"Seven is how many more than four? Write the answer in the blank."

Exercise 34.

"Four is how many less than nine? Write the answer in the blank."

Exercises 35 and 36.

"Put an X in the box in front of the correct answer to each question:

"When you put together things that are alike and find how many there are, you are _____?

"When you take some away from a group and find how many are left, you are _____?"

Exercises 37-41.

"In the left column there is some sign language. In the right column there are the meanings for these signs. In front of each sign on the left put the number that goes with its meaning." (Allow enough time for all to try.)

Exercise 42.

"Write the number that the dot picture represents in the blank."

Exercise 43.

"Fill in the blanks with the correct figures."

Exercises 44-52.

"Draw a ring around yes or no to show the correct answer:

"44. Will 8 pennies buy more than 1 dime?
"45. Does forty-nine mean 4 tens and 9 ones?
"46. Is 147 more than 174?
"47. Is 793 less than 397?
"48. Does 230 come after 229?
"49. Does 120 come before 119?
"50. Does 107 come between 106 and 108?
"51. Do these dot pictures show that we add tens the same way we add ones?
"52. Do these dot pictures show that we subtract tens the way we subtract ones?

Exercises 53-64.

"Jane had a good way of studying 13. She made dots like these, a group of ten and three more.

"By moving her pencil to the left, Jane could discover addition and subtraction facts about 13.

"Write the whole story as Jane wrote it, by filling in the blanks."

Exercises 65-67.

"Write the number that means:

8 tens (Pause.)
8 tens and 5 (Pause.)
30 and 1." (Pause.)

Exercises 68-70.

"Write the number names for these figures:

12 (Pause.)
90 (Pause.)
64." (Pause.)

Exercise 71.

"What number is written under the space where Wednesday (wed) should be? Write the number in the blank."

Exercise 72.

"Make an X in the box in front of the correct answer:
"How many inches are in 1 yard?

Exercise 73.

"Look at the lengths of these five lines. (Pause.) Which line is about three times as long as A? Write the answer in the blank."

Exercise 74.

"How many dimes have the same value as a half-dollar? Write the answer in the blank."

Exercises 75-78.

"Under each clock write the time that it shows."

Exercises 79-82.

"Look at the time under each clock. Make the hands of each clock show that time."

Exercises 83 and 84.

"Fill the blanks with the correct numbers."

TEST II

COMPUTATION

"Look on page 5 at Test II, Computation. Write the answers to these questions in addition and subtraction. One of each is already answered as a sample."

TEST III

PROBLEMS

"Look on page 6 at Test III, Problems. Here are some story problems. Read each one carefully. Write your answer on the blank at the right."

ARITHMETIC ACHIEVEMENT TEST

Ida Mae Heard, Associate Professor of Mathematics, Southwestern Louisiana Institute, Lafayette, Louisiana.

Name______________________________Grade_____Boy or girl_____

School____________________________Age___Birthday___________

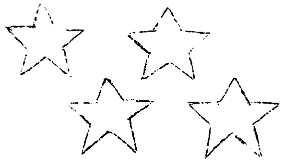
Teacher_________________________________Date_______________

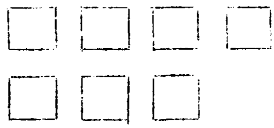
Test	Subject	Possible Score	Pupil's Score
I.	CONCEPTS AND BACKGROUND	[84]	_____
	A. Understanding the numbers 1-10	36	_____
	B. Signs and symbols	5	_____
	C. Understanding 2-place numbers	23	_____
	D. Understanding 3-place numbers	7	_____
	E. Measurement	13	_____
II.	COMPUTATION	[45]	
	A. Addition -- 28		
	Sums 1 to 10	4	_____
	Sums 11 to 18	9	_____
	Tens and ones (no carrying)	6	_____
	Column addition		
	With 3 addends, sums 1 to 10	4	_____
	With 3 addends, sums 11 to 18	3	_____
	With 3 addends, tens and ones	2	_____
	B. Subtraction -- 17		
	Minuends not greater than 10	5	_____
	Minuends not greater than 18	7	_____
	Tens and ones (no changing tens to ones)	5	_____
III.	PROBLEMS	[18]	
	A. Addition -- 10		
	Sums not greater than 10	4	_____
	Sums 11 to 18	2	_____
	Tens and ones (no carrying)	2	_____
	Column addition (no carrying)	2	_____
	B. Subtraction -- 8		
	Minuends not greater than 10	3	_____
	Minuends not greater than 18	2	_____
	Tens and ones (no changing tens to ones)	3	_____
	Total	147	

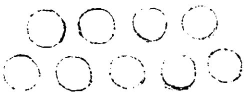
ARITHMETIC ACHIEVEMENT TEST

TEST I

CONCEPTS AND BACKGROUND

1.  3 4 5

2.  5 6 7

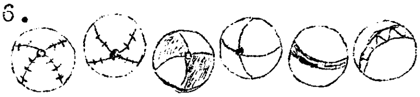
3.  8 9 10

4. 

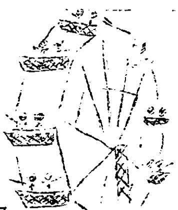
I see _____ boxes.

5. 

I see _____ trees.

6. 

I see _____ balls.



Jane	Jim	Jack	Ann	Tom	Fred	Sue	May	Joe	Lou

7. ________ is first in line for the ferris wheel.

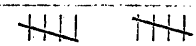
8. Jack is ________ in line.

9. ________ is eighth in line.

10. Lou is ________ to get the cotton candy.

11. ________ is fourth.

12. ________ is last.

13. Halloween Party  _____

14. Christmas Party  _____

15. Easter Egg Hunt  _____

Black and Yellow Chickens

16. 8 + ___ = 9
17. 7 + ___ = 9
18. 6 + ___ = 9
19. 5 + ___ = 9
20. 4 + ___ = 9
21. 3 + ___ = 9
22. 2 + ___ = 9
23. 1 + ___ = 9

Ran Away Left

24. 9 - 1 = ___
25. 9 - 2 = ___
26. 9 - 3 = ___
27. 9 - 4 = ___
28. 9 - 5 = ___
29. 9 - 6 = ___
30. 9 - 7 = ___
31. 9 - 8 = ___
32. 9 - 9 = ___

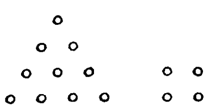
33. Seven is how many more than four? _____

34. Four is how many less than nine? _____

35. [ ] adding [ ] subtracting [ ] dividing [ ] multiplying

36. [ ] adding [ ] subtracting [ ] dividing [ ] multiplying

Sign Language		The Meaning
37.	___ +	1. take away, less
38.	___ x	2. cents
39.	___ -	3. and
40.	___ ¢	4. times
41.	___ $	5. dollars and cents
		6. divide
		7. dollars

42.  _____

43. Thirty-five is ___ tens and ___ ones.

44. Will 8 pennies buy more than 1 dime? Yes No

45. Does forty-nine mean 4 tens and 9 ones? Yes No

46. Is 147 more than 174? Yes No

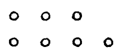
47. Is 793 less than 397? Yes No

48. Does 230 come after 229? Yes No

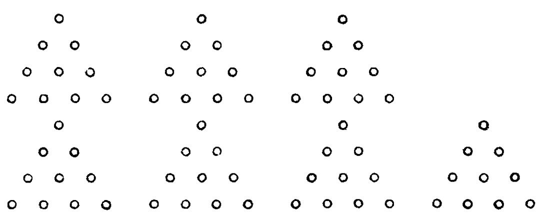
49. Does 120 come before 119? Yes No

50. Does 107 come between 106 and 108? Yes No

51. Do these dot pictures show that we add tens the same way we add ones?



3 ones
+4 ones
7 ones

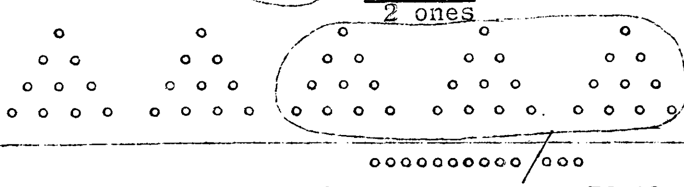


3 tens
+4 tens
7 tens

Yes No

52. Do these dot pictures show that we subtract tens the way we subtract ones?

5 ones
-3 ones
2 ones



5 tens
-3 tens
2 tens

Yes No

If 10 + 3 = 13

53. then 9 + ___ = 13
54. 8 + ___ = 13
55. 7 + ___ = 13
56. 6 + ___ = 13
57. 5 + ___ = 13
58. 4 + ___ = 13

If 13 - 3 = 10

59. then 13 - 4 = ___
60. 13 - 5 = ___
61. 13 - 6 = ___
62. 13 - 7 = ___
63. 13 - 8 = ___
64. 13 - 9 = ___

65. 8 tens ____

66. 8 tens and 5 ____

67. 30 and 1 ____

68. 12 ____

69. 90 ____

70. 64 ____

71. February

Sun	Mon					Sat
1	2	3	4	5	6	7

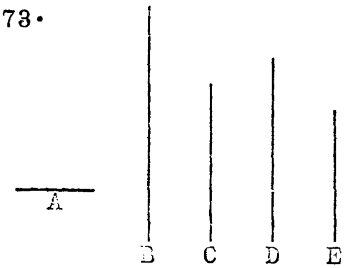
____

72.

[ ] 24 [ ] 3

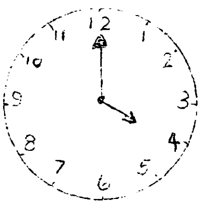
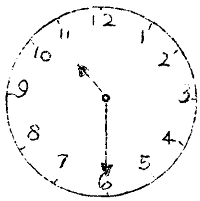
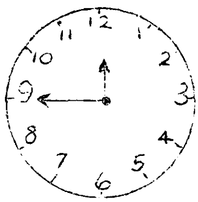
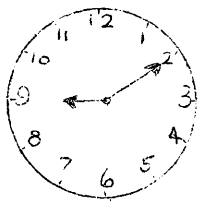
[ ] 12 [ ] 36

73.

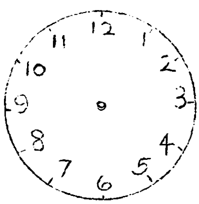
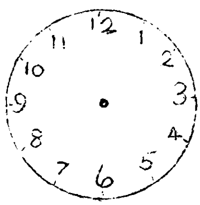
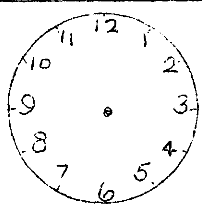
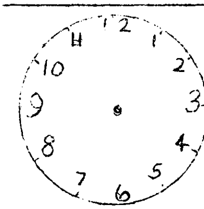


____

74. How many dimes have the same value as a half-dollar? ____

75. ____ 76. ____ 77. ____ 78. ____

79. 5 80. 15 to 6 81. 30 min. after 9 82. 5 min. after 4

83. 1 hundred, 0 tens, and 2 ones are ____.

84. 583 means ____ hundreds, ____ tens and ____ ones.

TEST II

COMPUTATION

Add:

Sample	(1)	(2)	(3)	(4)	(5)	(6)	(7)
2	2	5	4	7	9	8	5
+1	+3	+5	+4	+1	+2	+5	+6
3							

(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
7	6	4	9	9	8	12	62
+8	+9	+7	+9	+7	+6	+41	+15

(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
25	32	59	92	4	4	2	3
+21	+25	+40	+13	4	3	3	3
				1	3	5	2

(24)	(25)	(26)	(27)	(28)
5	7	4	73	81
2	1	2	10	13
6	8	6	65	35

Subtract:

Sample	(29)	(30)	(31)	(32)	(33)	(34)	(35)
6	8	7	8	4	10	15	11
-3	-5	-2	-6	-1	-6	-6	-3
3							

(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)
13	15	14	16	17	85	48	76
-8	-9	-6	-8	-9	-11	-24	-65

(44)	(45)
85	77
-73	-51

TEST III

PROBLEMS

1. Ann has 10 pennies. Jane has 8 pennies. How many fewer pennies has Jane than Ann? ______

2. Jack had 2 marbles in the ring when Joe came to play with him. Joe put 4 marbles in the ring. Then how many marbles were in the ring? ______

3. Jim caught 2 fish and his father caught 6 fish. How many fish did Jim and his father catch? ______

4. Joe has made 3 Christmas cards and Henry has made 4 Christmas cards. How many cards have they both made? ______

5. The girls brought their dolls to school. Alice brought 3 dolls and Jane brought 7 dolls. How many dolls did they bring? ______

6. There were six red birds in a tree. Three flew away. How many birds were left? ______

7. Ann read 6 stories and Jack read 1 story. Ann read how many more stories than Jack? ______

8. The Smith family got 9 Christmas cards one day and 8 cards the next day. How many cards did they get? ______

9. Tom worked 4 hours one week, and he worked 7 hours the next week. How many hours did Tom work? ______

10. Betty wants a doll that costs 15¢. She has 8¢. How much more money does she need? ______

11. Bob has 16 books, and Joe has 8 books. How many fewer books does Joe have than Bob? ______

12. Sue needs 10 cents for a bus ticket and 50 cents for a new doll. How much does she need? ______

13. Jack had 43 marbles. He bought 32 more. How many marbles had Jack then? ______

14. A toy car costs 20¢ and a toy train costs 80¢. How much cheaper is the car than the train? ______

15. Ann had 66 cents to spend. She paid 40 cents for a doll. How many cents did she then have? _____

16. Charles has 65 cents in his bank. How many cents will be left if he takes out 25 cents for a defense stamp? _____

17. Betty gathered eggs for her grandmother. She found 6 eggs in one nest, 3 eggs in another nest, and 4 eggs in still another nest. How many eggs did Betty find in the three nests? _____

18. Alice cut out 34 pictures of birds, 21 pictures of flowers, and 32 pictures of dolls. She put the pictures in a scrapbook. How many pictures did she put in the book? _____

AUTOBIOGRAPHY

Ida Mae Heard was born Ida Mae Pou on a farm near Tenaha, Texas, on April 4, 1910. The first three years of her elementary education were obtained in a nearby two-teacher rural school. In 1920 her family moved to Marshall, Texas, where she completed elementary and secondary school, and junior college.

She was married to William Allen Heard in 1929 and has two children, Ranse Allen and Betty Ruth. She received the Bachelor of Science Degree from North Texas State College in 1938. In 1943 she was granted the Master of Arts Degree from Teachers College, Columbia University, with a major in The Teaching and Supervision of Mathematics.

Mrs. Heard's eighteen years of teaching experience have included five years in grades one through six, five years in grades seven through nine, two years in high school, all in the public schools in Texas and New Jersey. In 1947 she came to Southwestern Louisiana Institute, at Lafayette, Louisiana, as Assistant Professor of Mathematics, and at the present writing is Associate Professor.

Mrs. Heard has been active in the work of the National Council of Teachers of Mathematics. She served as a member of the Board of Directors of this organization from 1950 to 1952, and has contributed to the organization's Twenty-

Second Yearbook. During the past five years she has engaged in research to improve instruction in arithmetic and has served as a consultant on the teaching of arithmetic in workshops throughout the state, in Texas, and in North Carolina.

EXAMINATION AND THESIS REPORT

Candidate: **Ida Mae Heard**

Major Field: **Education**

Title of Thesis: **The Use of Manipulative Materials in Teaching Arithmetic in Grade Three**

Approved:

George H. Sheer
Major Professor and Chairman

Richard J. Russell
Dean of the Graduate School

EXAMINING COMMITTEE:

W. A. Lawrence

Howard Hill

Marion B. Smith

E. B. Robert

Date of Examination:

May 6, 1954