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Localization of gravity in brane world cosmologies

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The most remarkable and interesting feature of brane world scenario is the use of bulk's curvature to localize gravity on the brane, albeit with fine tuning of the brane and bulk parameters. For FRW expanding universe on the brane, it is a moving hypersurface in Schwarzschild anti de Sitter bulk spacetime. We show that zero mass gravitons have bound state on the brane for suitable values of brane and bulk parameters. There occur various cases giving rise to different cosmological models, in particular we discuss a model with positive cosmological constant on the brane.

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The view that our Universe might actually have dimensions more than four is anchored on the recent developments in string and M-theories in which gravity arises as a truly higher dimensional interaction. Only in the low energy limit it manifests in the familiar 4-dimensional general relativity (GR). This has initiated a lot of activity in recent times. Though the basic idea was already there in the form of Kaluza-Klein (KK) theories, the recent spurt is primarily due to the possibility which helps solve the mass hierarchy problem in the standard model of particle physics.

In some of these models [1] the extra dimensions can be as large as millimeter which is however less than the current observational limits on low scale gravity. Notable are the models in which the extra dimensions are allowed to be of infinite extent [2, 3]. These models have Z_2 -symmetry, which is motivated by the reduction of M theory to $E_8 \times E_8$ heterotic string theory [4]. The single brane Randall-Sundrum (RS) model [3] has attracted a lot of interest and activity. In this model the Minkowski flat brane in 5-D anti de Sitter (AdS) bulk has a positive tension. It is then possible to recover Newton's inverse square law with r^{-4} correction term which arises from massive KK modes contribution. There have been various generalizations of this [5] in the form of thick branes [6], AdS branes [7] and brane models without Z_2 symmetry [8].

In the overall view of the brane world scenario, all matter fields live on the 3-brane which is the 4-D physical Universe while gravity can propagate in the extra dimensions, say 5-D bulk. The bulk and brane spacetimes are joined together through the Israel junction conditions [9]. Since, the standard Einstein equations on the brane are modified by the bulk effects it opens a whole new vista for investigation of astrophysical and cosmological consequences of these models (see for eg. [10]). The connection with CFT correspondence has also been studied (see for eg. [11]). For complete solution of the problem, one has to solve the Λ -vacuum equation in 5-D for the

bulk spacetime and simultaneously the modified Einstein equation which in addition to the square of stress tensor also contains the projection of the bulk Weyl curvature on the brane (dark radiation) and then the two solutions should be joined together with the Israel junction conditions [9]. It is by all means a very formidable task and it is therefore no surprise that there exist only few complete solutions to date. The two important cases are flat/vacuum brane with AdS bulk and FRW brane with Schwarzschild-AdS (S-AdS) bulk [12–14].

The S-AdS bulk-brane system is composed of two patches of S-AdS bulk having in general different mass parameters for the black hole with the brane located at $y = y(\tau)$, where $y(\tau)$ is determined by solving the Israel junction conditions [13]. From the resulting equation for brane trajectory one finds that in S-AdS bulk, the FRW brane will in general be moving unless the parameters are properly fine tuned (eg. for the RS case $\sigma = (3/4\pi G_5 l)$, $\Lambda_4 = 0$, where σ is the brane tension, G_5 is the 5-D gravitational constant and l is the radius of curvature of the bulk spacetime). The extra dimension is a radial coordinate of the bulk and imposing Z_2 symmetry across the brane demands that the mass parameters of both the patches to be same. By fine tuning parameters one can obtain static branes which cannot harbour expansion which is essential for realistic cosmological models. A slight off tuned value of σ or non zero value of black hole mass would set it moving.

Though localization of gravity for the AdS bulk with flat brane and for some generalizations of it has been established [7, 15, 16], it has not yet been investigated for the S-AdS bulk with FRW brane. The problem becomes difficult if one notes that unlike the RS case, the brane is dynamic in the static bulk and it is non trivial to fix the boundary conditions on the modes. For localizability of gravity on the brane, first there should exist bound normalizable mode for zero mass graviton and plus there should also exist KK modes to give the correction to the Newtonian gravity. All this could be studied by considering perturbation of the bulk metric and the brane motion determined by the Israel junction conditions. This is what we shall concern ourselves in this investigation which should be quite indicative of localiza-

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tion of gravity on the brane. For actual demonstration of recovering Newtonian gravity with the correction, one has to do propagator analysis which is hard to carry out for a non-static brane and we shall not attempt that here.

It is well known that localizability is very sensitive to fine tuning of parameters. A priori, there is no well established criterion to check this. For instance there does exist a bulk spacetime, which is an exact solution of the Λ -vacuum equation, for which gravity is non localizable on the brane [17]. This is the case of Nariai metric which has non zero Weyl curvature. Note that Weyl is non zero for S-AdS as well. It is therefore pertinent to find under what conditions do zero mass gravitons have bound state on FRW brane? This is the most critical question for brane world cosmologies and that is what we wish to address in this letter.

Since our brane is a hypersurface around the black hole, any movement of the brane towards or away from the black hole would be interpreted as contraction or expansion by the observers on the brane. That is how expansion is directly related to motion of the brane. To see whether zero mass graviton has a bound normalizable state in these cosmological models one has to solve for the graviton perturbation equation by taking into account the location of the brane. Unlike the static cases (like the RS case) the position of the FRW brane would not be fixed in the bulk and one would have to take into account the trajectory of the brane to understand the localizability. The brane trajectory found through Israel matching conditions would be given by a Friedmann like equation which determines the position of the brane. In the following we shall hence study the metric perturbations of the S-AdS bulk and obtain a potential in the graviton wave equation which would determine the fate of localizability once the brane trajectory (and hence the location of the brane) is taken care of. We shall then show how the RS case can be recovered in this scenario, then we shall in particular consider the case of $k = 1, \Lambda_4 > 0$, while a comprehensive discussion of all possible cases would be done elsewhere [18].

In the five dimensional bulk we have the S-AdS metric,

$$ds^2 = -e^{2\beta} dt^2 + e^{-2\beta} dy^2 + y^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right] \quad (1)$$

where

$$e^{2\beta} = \left(k + \frac{y^2}{l^2} - \frac{M}{y^2} \right). \quad (2)$$

Here $k = 0, \pm 1$ is the curvature index and M is the mass parameter of the bulk black hole.

The static limit would be given by $g_{00} = 0$ which leads to,

$$y_h^2 = \frac{l^2}{2} \left(-k + \sqrt{k^2 + \frac{4M}{l^2}} \right) \quad (3)$$

and since the metric is spherically symmetric this also defines the location of event horizon of the spacetime. The

above metric is the solution of the Λ -vacuum equation,

$$G_{ab} = -\Lambda_5 g_{ab}, \quad \Lambda_5 = -6/l^2. \quad (4)$$

The Latin indices which label the bulk spacetime (x^μ, y) run from 0...4 and the Greek indices labelling brane spacetime (x^μ) would run from 0...3. We consider the metric perturbations of the above metric $g_{ab}^{(B)}$, i.e. $h_{ab} = g_{ab} - g_{ab}^{(B)}$. We would take the metric perturbations in the extra dimension to vanish, $h_{ty} = h_{ry} = h_{\theta y} = h_{\phi y} = h_{yy} = 0$. We would further impose the transverse-traceless gauge conditions, $\nabla^\mu h_{\mu\nu} = 0, h^\mu{}_\mu = 0$ and hence the wave equation turns out to be,

$$\square_5 h_{ab} + 2 R_{cabd}^{(B)} h^{cd} - R_{ac}^{(B)} h_b{}^c - R_{bc}^{(B)} h_a{}^c = \Lambda_5 h_{ab}. \quad (5)$$

Assuming that wavelength of the gravitons is much smaller than the radius of curvature of the background and their amplitude is very small we can work under linearized approximation. We can further choose a constant vector field which satisfies $h_{abu}{}^b = 0$ and hence we are effectively left with two independent modes. We solve this wave equation with the ansatz that $h_{ab}(x^\mu, y) = h_{ab}(x^\mu) \Psi(y)$. Substituting this in eq.(5) and using m^2 as the constant of separation of variables, the y dependence turns out to be

$$\left(\frac{y^2}{l^2} - \frac{M}{y^2} + k \right) \Psi'' + \left(\frac{3M}{y^3} - \frac{k}{y} + \frac{y}{l^2} \right) \Psi' - \frac{4M}{y^4} \Psi + m^2 \Psi = 0 \quad (6)$$

where prime denotes a derivative with respect to y . This equation can be written down in the form

$$\Psi'' + a_1(y) \Psi' + a_2(y) \Psi = 0 \quad (7)$$

which can be transformed into a wave equation form with the transformation $\Psi(y) = \phi(y) \psi(y)$ where

$$\phi(y) = c \exp \left(-\frac{1}{2} \int_{y_i}^{y_f} a_1(y) dy \right). \quad (8)$$

Here c is a constant and y_i and y_f belong to the interval over which a_1, a_2 are continuous and a_1 has a continuous derivative. This transformation eliminates the first order derivative term in eq.(6) and we get the desired Schroedinger like equation

$$-\frac{1}{2} \psi(y)'' + V \psi(y) = m^2 \psi(y). \quad (9)$$

The potential consists of two parts, $V = V_f + V_m$, which are given by

$$V_f = - \frac{6l^2(5My^4 - ky^6)}{8(y^5 + l^2(-My + ky^3))^2} + \frac{2Ml^2}{y^2(y^4 - Ml^2 + kl^2y^2)} - \frac{y^8 + l^4(-15M^2 + 22kMy^2 - 3k^2y^4)}{8(y^5 + l^2(-My + ky^3))^2} \quad (10)$$

and the interaction part

$$V_m = - \frac{m^2 l^2 y^2 - 2m^2 (y^4 - Ml^2 + kl^2 y^2)}{2(y^4 - Ml^2 + kl^2 y^2)}. \quad (11)$$

The presence of interaction part is peculiar to the coordinate system we are working with. Obviously, the shape of the potential will play determining role in localization of gravitons. The potential V for all the cases provides a negative infinite well at the event horizon of the black hole of mass M , which holds irrespective of the matter distribution on the brane. The potential for the massless and massive modes for the case $k = 1, M \neq 0$ is shown in Fig. 1. The potential for the massless mode can be viewed as the $m \rightarrow 0$ limit of the potential for massive modes for which $V > 0$ for large y . Such a behaviour is a generic feature of various cosmological scenarios.

The Israel junction conditions determine the relation between the bulk and brane parameters,

$$\Lambda_4 = \frac{\Lambda_5}{2} + \frac{16\pi^2}{3} G_5^2 \sigma^2, \quad G_4 = \frac{4\pi}{3} G_5^2 \sigma \quad (12)$$

and also the motion of the brane given by the equation [13],

$$\dot{y}^2 - \frac{8\pi G_4}{3} \rho \left(1 + \frac{\rho}{2\sigma}\right) y^2 + (1 - \sigma^2/\sigma_0^2) \frac{y^2}{l^2} - \frac{M}{y^2} + k = 0. \quad (13)$$

Here dot refers to derivative relative to proper time τ and ρ is the energy density on the brane and the critical tension $\sigma_0 \equiv 3/4\pi G_5 l$, which determines the sign of Λ_4 . The y coordinate now plays a dual role. It not only tells us about the effect on graviton fluctuations due to extra dimension but also parameterizes the brane trajectory. The metric on the brane is FRW and is given by,

$$ds_4^2 = -d\tau^2 + y^2(\tau) \left(\frac{dr^2}{1 - kr^2} + d\Omega_2^2 \right) \quad (14)$$

where $d\tau^2 = \exp(2\beta(t))(1 - \exp(-4\beta(t))(dy/dt)^2)dt^2$. The physically interesting branes would lie outside the horizon which would also serve as the high energy cut-off. For localization of massless mode we require it to be bounded as well as normalizable. The form of the potential for various choice of cosmological parameters is such that boundedness of massless mode is guaranteed once the brane crosses horizon. However, the normalizability of the massless mode depends on the asymptotic behavior of the potential and the form of the potential reflects that branes which are either static or ever expanding in the bulk would harbour localization.

As is clear from eq. (13) that motion of brane is determined by both black hole mass and energy distribution on the brane. The localization would therefore depend on energy distribution in both bulk and brane. The localization of the massless mode depends on whether the brane expands forever or has a bounce, and it can be easily checked from eq.(13) that such a behaviour of the brane trajectory is unchanged even if the matter density terms are excluded. For simplicity we would henceforth exclude matter density terms from our analysis of brane trajectory and hence the brane trajectory equation becomes

$$\dot{y}^2 - \frac{\Lambda_4}{3} y^2 - \frac{M}{y^2} + k = 0. \quad (15)$$

Note that this equation is non linear in the highest order of derivative and hence it will not have unique solution.

It should be noted for dynamic branes the proper time as measured by the observers on the brane is not the same as the time component of the bulk metric. For the observer on the brane to interpret the massless mode correctly one thus has to assume that $y(t)$ is a slowly varying function. Further, fixing boundary conditions for the modes on a moving boundary is a very difficult task. However, in a cosmological setting of FRW branes we can safely assume that brane is slowly expanding and these problems can be bypassed. Our results for dynamic branes would hence hold in the above cosmological scenario.

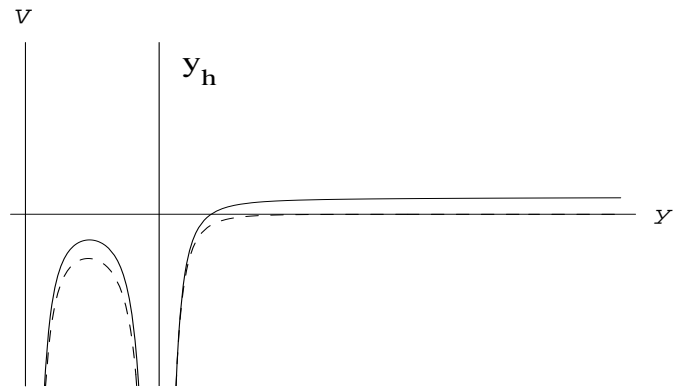


FIG. 1: Potential plot for $M \neq 0$ and $k = 1$. The dashed and dark lines indicate potential for massless and massive mode respectively.

Now we would see how the above metric perturbation equation and the brane dynamics equation yield RS model. For this $k = M = 0, \sigma = \sigma_0$, the metric eq.(1) takes the form

$$ds^2 = -\frac{y^2}{l^2} dt^2 + \frac{l^2}{y^2} dy^2 + y^2 (dr^2 + r^2 dr^2 + r^2 \sin^2 \theta d\phi^2). \quad (16)$$

Using the transformation $\eta = l \ln(l/y)$ one can rewrite this metric in the familiar RS form. Note that y is a radial coordinate of the black hole in the bulk and it ranges from 0 to ∞ . The above transformation would yield η ranging from $-\infty$ to ∞ . Imposition of Z_2 symmetry across the brane (i.e. matching the extrinsic curvatures on both sides of the brane) does not give any condition on y coordinate, however in η coordinate it demands $\eta_{(-)}$ to be identified with $\eta_{(+)}$ and hence the metric contains $|\eta|$ in the warp factor,

$$ds^2 = d\eta^2 + e^{-2|\eta|/l} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2). \quad (17)$$

RS brane is static and located at $\eta = 0$, which means at $y = l$. Note that the shape and form of the potential in the Schroedinger equation critically depends on the coordinates one is working with. In the η coordinates it is the presence of $|\eta|$ which gives a Dirac delta

potential at the location of the brane. However, working with the y coordinate one gets from eqs.(10) & (11), $V = -1/8y^2 + m^2(1 - l^2/2y^2)$. At the location of the brane the massless mode is bound and the asymptotic behaviour of the potential suggests that it is normalizable. The massive modes for which $m > 1/2l$, $V > 0$, are unbound. Solving near the brane one can obtain the wavefunctions from eq.(9) which turn out to be similar to those obtained by RS, with the required ml suppression of KK modes on the brane [18]. The fact that V turns positive for massive modes gives the continuum spectrum and the RS correction to Newtonian potential. The form of the potential also suggests that there exist discrete modes for $m < 1/2l$. The appearance of discrete modes in this case is due to our choice of the coordinates. This is clear from the fact that unlike RS coordinates our extra dimension is a radial coordinate in the bulk and our brane is curved. However, it should be noted that the correction to Newtonian gravity is not expected to change since the Green's function used by Garriga and Tanaka [19] to evaluate the force law between two point sources on the brane in the RS case is inert under our transformation of coordinates from y to η . In the y coordinates their Green's function can be written as

$$G_R(x, x') = - \int \frac{d^4 k}{(2\pi)^4} e^{ik_\mu(x^\mu - x'^\mu)} \left[\frac{(y^2/l^2)(y'^2/l^2)}{l(\mathbf{k}^2 - (w + i\epsilon)^2)} + \int_0^\infty \frac{u_m(y)u_m(y') dm}{m^2 + \mathbf{k}^2 - (w + i\epsilon)^2} \right] \quad (18)$$

where $u_m(y)$ are the wavefunctions for massive modes. In the stationary case the Green's function between two points at the location of the brane, i.e. $y = y' = l$ yields,

$$G(x^i, l, x'^i, l) \approx -\frac{1}{4\pi l r} \left(1 + \frac{l^2}{2r^2} \right) \quad (19)$$

which leads to the same l^2/r^2 correction to Newtonian gravity.

For negative Λ_4 branes the brane trajectory exhibits a bounce, like for the case $k = -1, M = 0, \Lambda_4 < 0$ it is given by $y(\tau) = \sin(\sqrt{-\Lambda_4/3}\tau)/\sqrt{-\Lambda_4/3}$. The potential is similar as in RS case, apart from that it blows up at origin and there is an infinite negative well at $y = l$, but the brane trajectory is such that it does not yield the required behaviour of the potential for the ground state wavefunction to be normalizable. Hence the massless mode would not be localized for the negative Λ_4 brane (but that would be for the positive Λ_4 brane because the solution of the brane motion equation does permit normalized wavefunction). We thus recover the well-known result for AdS bulk and negative/positive Λ_4 branes [7]. The behavior of brane trajectory also plays critical role in localization of gravity on the brane.

Now we turn to FRW brane. Eq.(15) yields host of solutions for interesting cosmological scenarios including the inflationary solutions for all values of k with a positive Λ_4 on the brane, which is also favored by current

observations of type Ia supernovae [20]. A detailed discussion of all these cases would be done elsewhere [18], here we would as a representative consider the case of $k = 1, M \neq 0$ and $\Lambda_4 > 0$.

Solving eq.(15) for this case we get,

$$y(\tau) = \sqrt{\frac{3}{2\Lambda_4}} \left[1 + n \sinh(2x) - \cosh(2x) \right]^{1/2} \quad (20)$$

where $n = 2\sqrt{M\Lambda_4/3}$ and $x = \sqrt{\Lambda_4/3}\tau$. The behavior of potential V for this case is shown in Fig.1. Note that V blows up at $y = 0$ and there is an infinite well at $y = y_h$, we shall therefore restrict to $y > y_h$. Since the form of the potential for localization requires an ever expanding brane, only $n > 1$ is possible. The brane emerges out of the event horizon, expanding from $y = 0$ at $\tau = 0$ like a white hole [21] and would expand for ever, exponentially for large τ . The Hubble parameter for $k = 1, M \neq 0$ cosmological model is given by

$$H = \frac{\dot{y}}{y} = \sqrt{\frac{\Lambda_4}{3}} \frac{\left[n \cosh(2x) - \sinh(2x) \right]}{\left[1 + n \sinh(2x) - \cosh(2x) \right]} \quad (21)$$

implying an inflationary universe at large τ .

For the massless mode the potential profile favors boundedness and normalizability. The massive modes would be unbounded and would contribute to correction over Newtonian potential. In order to show the correction suggested by massive modes we would first parameterize the location of the brane off the horizon by $\alpha = 1 - M/y^2$. Solving the Schrodinger equation, eq.(9), in the approximation $M \ll l^2$ and near the horizon ($y_h \sim \sqrt{M}$) we get,

$$\psi(y) = \sqrt{y} (C_1 I_{-\gamma/2}(\nu y) + C_2 I_{\gamma/2}(\nu y)) \quad (22)$$

where $\gamma = \sqrt{1 - 4l^4(1 - 4\alpha)(M - 2\alpha l^2)/M^3}$ and $\nu = \sqrt{(1 - \alpha)(2\alpha l^2 - M)ml/M}$. The boundary conditions on the wavefunction at the brane in a cosmological setting leads to $C_1 = (ml)^{(1+\gamma)/2} M^{\gamma/4}$ with $\gamma < 2$ and $C_2 = 0$. Close to the horizon $\nu \sim ml/\sqrt{M}$ and since $y \sim \sqrt{M}$, the argument in the Bessel function as in the RS case is proportional to ml at the location of the brane. This suggests a l^2/r^2 correction to Newtonian potential due to massive modes.

The modifications to the standard GR would be most prevelant near the event horizon which is the high energy end and marks a cut off for the scale factor $y(\tau)$. As the brane moves out and expands, the potential as shown in Fig.1 becomes shallower and the high energy modifications die out with time. In particular, the RS correction to the Newtonian potential will die out as the universe expands.

The overall picture that emerges is that we could have different FRW cosmological models on the brane which are anchored onto S-AdS bulk. The curvature of the

bulk spacetime and the brane trajectory are key players in the localization of gravity on the brane. The ultimate evolutionary fate of models is decided by the black hole mass and the cosmological constant on the brane. We have thus shown that S-AdS bulk does allow localization of gravity on FRW brane with $\Lambda_4 > 0$.

In conclusion, we could say that brane cosmologies are firmly anchored in respect of localization of gravity. We have shown that the potential in the Schroedinger equation and the brane dynamics lead to bound state for zero

mass graviton on a slowly moving brane in a cosmological setting.

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