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Loop quantum gravity effects on inflation and the CMB

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In loop quantum cosmology, the universe avoids a big bang singularity and undergoes an early and short super-inflation phase. During super-inflation, non-perturbative quantum corrections to the dynamics drive an inflaton field up its potential hill, thus setting the initial conditions for standard inflation. We show that this effect can raise the inflaton high enough to achieve sufficient e-foldings in the standard inflation era. We analyze the cosmological perturbations generated when slow-roll is violated after super-inflation, and show that loop quantum effects can in principle leave an indirect signature on the largest scales in the CMB, with some loss of power and running of the spectral index.

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I. INTRODUCTION

The inflationary paradigm has been very successful, solving various problems in the big bang model of cosmology, and providing a framework for understanding structure formation in the Universe. Recent observational data on the cosmic microwave background (CMB) anisotropies, together with other data, are well accounted for by a simple inflationary model with suitable cold dark matter and dark energy content [1].

However, this simple picture belies a number of fundamental puzzles. In particular, there is the problem of how to explain the initial conditions for successful inflation; for example, with the simple potential,

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \,, \tag{1}$$

one requires an initial inflaton value $\phi_i \gtrsim 3M_{\rm pl}$. The "eternal inflation" scenario provides an answer to this, relying on the infinite extent of space, but its arguments have been challenged [2]. In any event, whatever the merits of eternal inflation, it is useful to seek alternative explanations and to explore what quantum gravity may tell us about this. Quantum gravity effects above the Planck energy $M_{\rm pl}$ should be able to set the initial conditions for inflation–or to provide an alternative to inflation with adequate predictive power for structure formation. This is one of the challenges facing candidate theories of quantum gravity.

Results from WMAP also point to a possible loss of power at the largest scales and running of the spectral index. These effects, if confirmed, are difficult to explain within standard slow-roll single-field inflation, and may have roots in Planck-scale physics.

Loop quantum gravity (or quantum geometry) is a candidate quantum gravity theory that has recently been applied to early universe cosmology (see [3] for a recent review). It improves on a conventional Wheeler-DeWitt quantization, by discretizing spacetime and thus avoiding the breakdown of the quantum evolution even when the classical volume becomes zero [4].

The quantization procedure involves a fiducial cell corresponding to a fiducial metric to define a symplectic structure. Introduction of a fiducial cell, which is not required at a classical level, is a necessity for Hamiltonian quantization. This procedure has been developed for spatially flat and closed Friedmann geometries (and related Bianchi geometries). It is important to note that the scale factor in the quantum regime has a different meaning than in classical general relativity [5]. For a flat geometry (which is the case we consider), in the quantum regime the scale factor is not subject to arbitrary rescaling, as in general relativity. Instead, the scale factor $\tilde{a}$, obtained from loop quantization by redefining canonical variables, is related to the conventional scale factor a via

$$\tilde{a}^3 = a^3 V_0 \,, \tag{2}$$

where V_0 is the volume of the fiducial cell. In this definition the quantization is independent of the rescaling of the fiducial metric and thus invariant under rescaling of the conventional scale factor. However, one can also quantize without redefining the canonical variables. In this case it has been shown that states in Hilbert space corresponding to two different fiducial metrics are unitarily related and the physics does not change with rescaling of the fiducial metric [6]. This is consistent with the background-independence of loop quantization.

What emerges in the quantization is a fundamental length scale,

$$\tilde{a}_* = \ell_* \equiv \sqrt{\frac{\gamma j}{3}}\, \ell_{\rm pl} \,, \tag{3}$$

where $\gamma\,(\approx 0.13)$ is the Barbero-Immirzi parameter and j is a half-integer (where $j > 1$ is necessary in order to study evolution via an effective matter Hamiltonian [7]). This length scale corresponds to a particular effective quantum volume (or the state of the universe) below which the dynamics of the universe is significantly modified by loop quantum effects. The crucial quantity that

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determines the dynamics and quantifies the state of the universe relative to the critical state, is

$$q \equiv \frac{\tilde{a}^2}{\tilde{a}_*^2} = \frac{a^2}{a_*^2} \,. \tag{4}$$

The Planck length $\ell_{\rm pl}$ is not put in by hand, but arises from the quantization procedure. The fundamental scale ℓ_* is the same for a flat or closed classical geometry and has nothing to do with the topology. Thus for a flat non-compact spacetime, there is a fundamental length scale defined by quantization, unlike general relativity, where no length scale (other than the Hubble length) is defined by the geometry. If a compact topology is imposed on a flat geometry, then that introduces another (classical) length scale, independent of ℓ_*. Here we consider a flat non-compact universe. In the semi-classical regime, where spacetime is well approximated as a continuum but the dynamics is subject to non-perturbative loop quantum corrections, the conventional scale factor has the usual rescaling freedom. For convenience, we use the rescaling freedom to fix the classical scale factor at the critical epoch of transition from quantum to classical evolution:

$$a_* = \ell_* \,. \tag{5}$$

It should be noted that any other choice could be used, or the rescaling freedom could be kept. The point is that the relevant physical quantity q remains invariant.

II. NON-PERTURBATIVE SEMI-CLASSICAL DYNAMICS

In loop quantization, the geometrical density operator has eigenvalues [6, 7]

$$d_j(a) = D(q)\,\frac{1}{a^3}\,, \tag{6}$$

where the quantum correction factor for the density in the semi-classical regime is

$$D(q) = \left(\frac{8}{77}\right)^6 q^{3/2}\Big\{7\Big[(q+1)^{11/4} - |q-1|^{11/4}\Big] - 11q\Big[(q+1)^{7/4} - {\rm sgn}\,(q-1)|q-1|^{7/4}\Big]\Big\}^6. \tag{7}$$

In the classical limit we recover the expected behaviour of the density, while the quantum regime shows a radical departure from classical behaviour:

$$a \gg a_* \;\Rightarrow\; D \approx 1\,, \tag{8}$$

$$a \ll a_* \;\Rightarrow\; D \approx \left(\frac{12}{7}\right)^6\left(\frac{a}{a_*}\right)^{15}. \tag{9}$$

Then d_j remains finite as $a \to 0$, unlike in conventional quantum cosmology, thus evading the problem of the big-bang singularity. Intuitively, one can think of the modified behaviour as meaning that classical gravity, which is always attractive, becomes repulsive at small scales when quantized. This effect can produce a bounce where classically there would be a singularity, and can also provide a new mechanism for inflationary acceleration.

In loop quantum cosmology, a scalar field ϕ with potential $V(\phi)$ in a flat Friedmann-Robertson-Walker background is described by the Hamiltonian [7]

$$\mathcal{H} = a^3 V(\phi) + \frac{1}{2} d_j\, p_\phi^2\,,\; p_\phi = d_j^{-1}\dot{\phi}\,, \tag{10}$$

where p_ϕ is the momentum canonically conjugate to ϕ. This gives rise to an effective Friedmann equation for the Hubble rate $H = \dot{a}/a$,

$$H^2 = \frac{8\pi\ell_{\rm pl}^2}{3}\left[V(\phi) + \frac{1}{2D}\dot{\phi}^2\right], \tag{11}$$

together with the modified Klein-Gordon equation

$$\ddot{\phi} + \left(3H - \frac{\dot{D}}{D}\right)\dot{\phi} + DV_\phi(\phi) = 0\,. \tag{12}$$

The quantum corrected Raychaudhuri equation follows from Eqs. (11) and (12),

$$\dot{H} = -4\pi\ell_{\rm pl}^2\dot{\phi}^2\,\frac{1}{D}\left(1 - \frac{\dot{D}}{6HD}\right). \tag{13}$$

In the quantum limit, Eq. (9) shows that the kinetic terms dominate the potential terms in Eqs. (11) and (12), and this leads to:

$$\dot{\phi} \propto a^{12}\,, \quad a \propto (-\eta)^{-2/11}\,, \tag{14}$$

where η is conformal time. Since $\dot{D}/(HD) = 15$ for $a \ll a_*$, we have from Eq. (13)

$$\dot{H} \approx 6\pi\ell_{\rm pl}^2\frac{\dot{\phi}^2}{D} > 0\,. \tag{15}$$

Quantum effects thus drive super-inflationary expansion [7]. However, this can not yield sufficient e-folds to overcome the flatness and horizon problems in the absence of the inflaton potential.

In the quantum regime, $a \ll a_*$, when d_j behaves as a positive power of a, the second term on the left of Eq. (12) acts like an anti-friction term and pushes the inflaton up the hill. The strong dominance of the kinetic term over the potential term means not only that the mechanism is robust to a change in $V(\phi)$, but also that it dominates over gradient terms in ϕ.

III. INFLATION WITH LOOP QUANTUM MODIFICATIONS

With the growth in a, the eigenvalue d_j eventually behaves as in Eq. (8). The second term on the left of

Eq. (12) then behaves as a friction term, but it takes some time before this term halts the motion of ϕ, since the initial quantum push is very strong. Then the field begins to roll down the potential from its maximum value $\phi_{\max}$, initiating a standard slow-roll inflationary stage.

We assume the inflaton is initially at the minimum of its potential. Initial small quantum fluctuations are sufficient to start the process of raising the inflaton up the hill. Strong kinetic effects will rapidly overwhelm the quantum fluctuations. We assume these fluctuations are constrained by the uncertainty principle, $|\Delta\phi_i \Delta p_{\phi\, i}| > 1$, so that

$$|\phi_i \dot{\phi}_i| \gtrsim \frac{10^3}{j^{3/2}} \left(\frac{a_i}{a_*}\right)^{12} M_{\rm pl}^3 \,. \qquad (16)$$

We take $a_i = \sqrt{\gamma} l_{pl}$ (for $a_i \ll \ell_{\rm pl}$, space is discretized and we cannot use the smooth dynamical equations above). The sign of $\dot{\phi}_i$ determines whether the inflaton just moves further up the hill (for $\dot{\phi}_i > 0$) or returns to $\phi = 0$ and then is pushed up (for $\dot{\phi}_i < 0$).

In order to make this qualitative description more precise, consider the simple potential, Eq. (1). Large-scale CMB anisotropies require that [8]

$$\phi_{\max} \gtrsim 3M_{\rm pl}\,, \quad m_\phi \sim 10^{-6} M_{\rm pl}\,. \qquad (17)$$

We find that loop quantum effects can produce a large enough $\phi_{\max}$ even for very small $\phi_i/M_{\rm pl}$ and $\dot{\phi}_i/M_{\rm pl}^2$ [satisfying the uncertainty constraint, Eq. (16)]. The results are shown in Figs. 1 and 2.

In the quantum regime ($a < a_*$), the energy density of the universe is dominated by the kinetic energy of the inflaton. Since $D^{-1} \sim q^{-15/2} \gg 1$ for $q \ll 1$ in Eq. (11), the Hubble rate can take large values even if $\dot{\phi}^2$ is small. As seen in the inset of Fig. 1, the Hubble rate increases for $a \lesssim a_*$ due to the growth of the kinetic term in Eq. (11).

By Eq. (13), super-inflation ends when $\dot{D}/(6HD) = 1$. We find numerically that this term quickly goes to zero just after the Hubble maximum. Figure 1 also shows the short non-inflationary phase ($\ddot{a} < 0$) while the field continues rolling up. The second stage of inflation begins as the inflaton approaches its maximum ($\dot{\phi} = 0$). Since $d_j(a) \approx a^{-3}$ at this stage, this is standard chaotic inflation, followed by conventional reheating when the inflaton oscillates.

In Fig. 2 we plot the maximum value $\phi_{\max}$ reached via quantum gravity effects, for various values of j and $\dot{\phi}_i$. Sufficient inflation ($\gtrsim 60$ e-folds) requires $\phi_{\max} \gtrsim 3M_{\rm pl}$ and Fig. 2 shows this is possible for a wide range of parameters. An increase in j decreases q and $d_j(a)$, thereby yielding larger H and $\ddot{\phi}$, and so increasing the number of e-folds.

Loop quantum effects can therefore in principle set the initial conditions for successful slow-roll inflation. A further question is whether any observational signature of the first phase of quantum gravity inflation survives the second phase of classical inflation. It should be

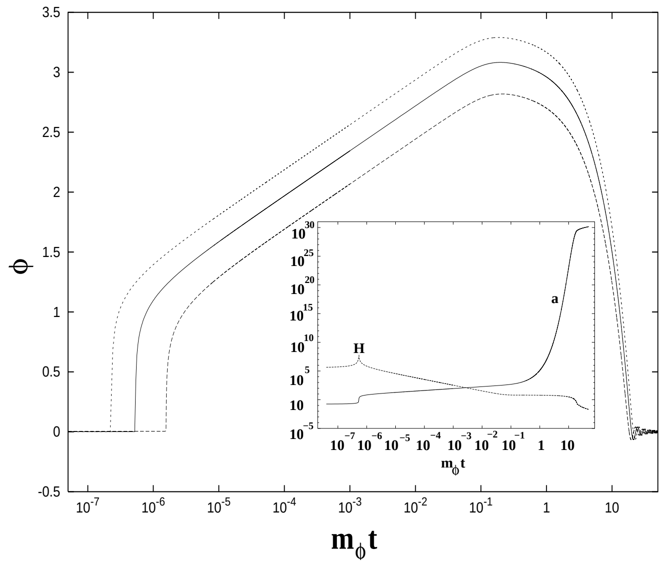

FIG. 1: Evolution of the inflaton (in Planck units). We set $\dot{\phi}_i/(m_\phi M_{\rm pl}) = 2$, with $m_\phi/M_{\rm pl} = 10^{-6}$, and choose $\phi_i/M_{\rm pl}$ as the minimum value satisfying the uncertainty bound, Eq. (16), i.e. $\phi_i/M_{\rm pl} \sim 10^{12} j^{-15/2}$. The solid curve has $j = 100$, the upper dashed curve has $j = 125$, and the lower dashed curve has $j = 75$.
Inset: Evolution of the scale factor and Hubble rate (in units of m_ϕ) with the same parameters as the solid curve for ϕ.

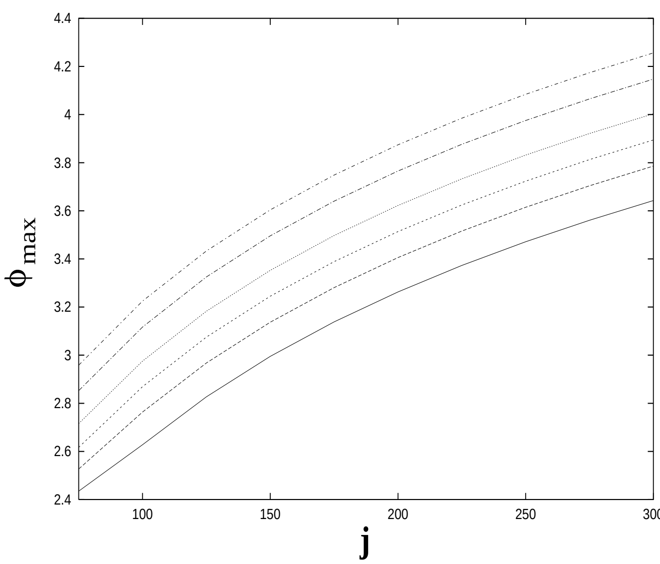

FIG. 2: The maximum reached by ϕ (in Planck units) as a function of j. From top to bottom, the curves correspond to initial conditions $\dot{\phi}_i/(m_\phi M_{\rm pl}) = 5, 2.5, 1, 0.5, 0.25, 0.1$ and $\phi_i/M_{\rm pl}$ is taken as the minimum value satisfying the uncertainty bound, Eq. (16). An increase (decrease) in j and $\dot{\phi}_i$ leads to an increase (decrease) in the number of e-folds.

noted that violation of slow roll occurs *after* the super-inflationary phase has ended and the universe is classical. In this sense, violation of slow roll is an *indirect* loop quantum gravity effect. The violation of the slow-roll condition around $\dot\phi = 0$, which is peculiar to this scenario, can lead to some suppression of the power spectrum at large scales and running of the spectral index, provided that

$$\phi_{\rm max} \sim \phi_{\rm ls}\,, \tag{18}$$

where $t_{\rm ls}$ is the time when the largest cosmological scales are generated. If $\phi_{\rm max} \gg \phi_{\rm ls}$, i.e. if quantum gravity effects drive the inflaton far up the hill, then cosmological scales are generated well into the classical era and there is *no* loop quantum signature in the currently observed CMB.

IV. CMB ANISOTROPIES

To be more concrete, we consider cosmological perturbations generated when the universe is in the classical regime ($a \gg a_*$), but slow-roll is violated. The spectrum of comoving curvature perturbations, $\mathcal{R}$, generated in slow-roll inflation is given by [8]

$$\mathcal{P}_{\mathcal{R}} \propto k^{n-1}\,, \quad n \approx 1 - 6\epsilon + 2\eta\,, \tag{19}$$

$$\epsilon \equiv \frac{M_{\rm pl}^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2\,, \quad \eta \equiv \frac{M_{\rm pl}^2}{8\pi}\frac{V_{\phi\phi}}{V}\,. \tag{20}$$

For the potential (1), this yields a slightly red-tilted spectrum,

$$n \approx 1 - \frac{1}{\pi}\left(\frac{M_{\rm pl}}{\phi}\right)^2\,. \tag{21}$$

The amplitude of the power spectrum generated in slow-roll inflation is [8] $\mathcal{P}_{\mathcal{R}} \approx H^4/\dot\phi^2$. In the loop quantum scenario, ϕ changes direction after it climbs up the potential hill to its maximum, $\dot\phi = 0$. It was shown in Ref. [9] that the spectrum of curvature perturbations is not divergent even at $\dot\phi = 0$, when the standard formula breaks down and should be replaced by

$$\mathcal{P}_{\mathcal{R}} \approx \frac{9H^6}{V_\phi^2} \approx \frac{9H^6}{m_\phi^4\phi^2}\,. \tag{22}$$

This appears to correspond to the replacement of $\dot\phi$ in the standard formula by the slow-roll velocity $\dot\phi = -V_\phi/(3H)$, but Eq. (22) is valid even at $\dot\phi = 0$, when the slow-roll approximation breaks down [9].

Using Eq. (22), the spectral index becomes

$$n = 1 + \frac{1}{1+\epsilon_1}\left(6\epsilon_1 - \frac{2\dot\phi}{H\phi}\right)\,, \tag{23}$$

where $\epsilon_1 \equiv \dot H/H^2$, which vanishes at $\dot\phi = 0$ and thus leads to a scale-invariant spectrum at the turn-over of the inflaton. This modification of the spectral index around $\dot\phi = 0$ is a distinct feature of the loop quantum scenario. The formula (23) reduces to the standard one in the slow-roll region ($|\ddot\phi| \ll |3H\dot\phi|$), since $\dot\phi = -V_\phi/(3H)$ and $|\epsilon_1| \ll 1$.

When the field climbs up the potential hill ($\dot\phi > 0$), we have $n < 1$ by Eq. (23). Therefore the spectrum begins to grow toward large scales for modes which exit the Hubble radius during the transient regime. However the power spectrum is close to scale-invariant as long as $\dot\phi$ is close to zero. We should also mention that there exists a short non-inflationary phase after the Hubble maximum (see Fig. 1). However the fluctuations in this non-inflationary phase are not of relevance, since the standard causal mechanism for the generation of perturbations does not operate.

The spectral index can be expanded as

$$n(k) = n(k_0) + \frac{\alpha(k_0)}{2}\ln\left(\frac{k}{k_0}\right) + \cdots\,, \tag{24}$$

$$\alpha = \left(\frac{{\rm d}n}{{\rm d}\ln k}\right)_{k=aH}\,, \tag{25}$$

where k_0 is some pivot wavenumber. In our case, the spectral index changes rapidly around the region $\dot\phi = 0$, which leads to larger running compared to the slow-roll regime. This means that it is not a good approximation to use a constant running in the whole range of the potential-driven inflation. For example one has $n \sim 0.964$ and $\alpha \sim -0.005$ at $\phi = 3M_{\rm pl}$ from Eq. (21). We find numerically that the running becomes stronger around $\dot\phi = 0$, with minimum value $\alpha \sim -0.06$.

Consider a scenario in which the spectral index grows rapidly towards 1 around $\phi = 3M_{\rm pl}$, corresponding to the pivot scale $k_0 \sim 10^{-3}\,{\rm Mpc}^{-1}$. One can express n in this region in terms of the average value $\bar\alpha$, i.e., $n(k) \approx n(k_0) + (\bar\alpha/2)\ln(k/k_0)$. Then we obtain $n(k) \sim 1$ at $k \sim 0.1k_0$ for $\bar\alpha = -0.04$. This behavior was found numerically, and leads to a larger spectral index around the scale $k \sim 10^{-4}\,{\rm Mpc}^{-1}$, compared to the standard slow-roll chaotic inflationary scenario.

In Fig. 3 the CMB power spectrum is plotted for several values of $\bar\alpha$ for $k \le k_0$, and with the average running $\bar\alpha = -0.005$ for $k > k_0$. In standard chaotic inflation $|\alpha| \lesssim 0.01$ around $\phi \sim 3M_{\rm pl}$, in which case it is difficult to explain the running with $\alpha = -0.077^{+0.050}_{-0.052}$ around the scale $k \sim 10^{-3}\,{\rm Mpc}^{-1}$ reported by the WMAP team [1]. In the loop quantum inflation scenario, it is possible to explain this running of the power spectrum due to the existence of the non-slow-roll region ($\dot\phi \sim 0$) that follows loop quantum inflation–provided that Eq. (18) holds, i.e., the loop quantum inflation does not push the inflaton too high up its potential hill. If the e-folds in slow-roll inflation are greater than 60, then the CMB power spectrum does not carry a loop quantum signature [10]. Although strong suppression of power around the multipoles $l = 2, 3$ is difficult to obtain unless $\bar\alpha \lesssim -0.3$, it is quite intriguing that the loop quantum scenario can

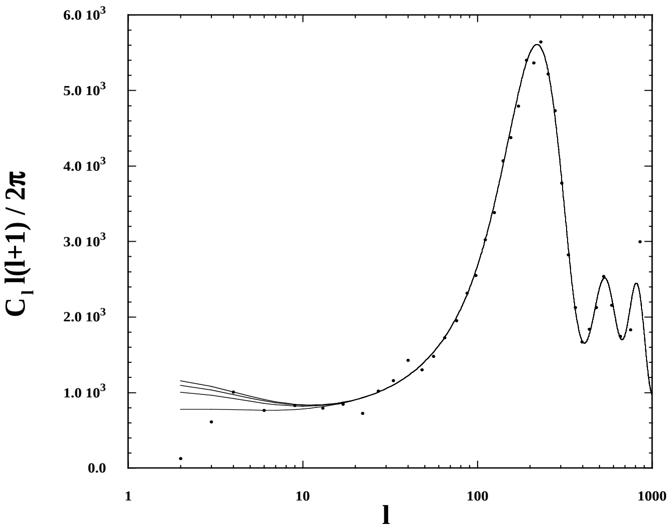

FIG. 3: The CMB angular power spectrum with loop quantum inflation effects. From top to bottom, the curves correspond to (i) no loop quantum era (standard slow-roll chaotic inflation), (ii) $\bar{\alpha} = -0.04$ for $k \leq k_0 = 2 \times 10^{-3}\,\mathrm{Mpc}^{-1}$, (iii) $\bar{\alpha} = -0.1$ for $k \leq k_0$, and (iv) $\bar{\alpha} = -0.3$ for $k \leq k_0$. There is some suppression of power on large scales due to the running of the spectral index.

provide a possible way to explain the observationally supported running of the spectrum, with some loss of power on large scales, albeit with fine-tuning of parameters.

V. CONCLUSIONS

In summary, we have shown that loop quantum effects can drive the inflaton from equilibrium up its potential hill, and can thus in principle set appropriate initial conditions for successful standard inflation. Loop effects may also leave an indirect imprint on the CMB on the largest scales, if the inflaton is not driven too far up its potential hill. This is possible in particular for very small initial fluctuations in the inflaton and its velocity, compatible with the uncertainty principle, Eq. (16). We stress that the observational signature we found is an indirect effect of loop quantum gravity, rather than a direct consequence of perturbations generated in the regime $a < a_*$. In the loop quantum super-inflationary era, the standard quantization using the Bunch-Davies vacuum is no longer valid. Further analysis is required to understand the quantization of metric perturbations in loop quantum gravity.

By analyzing the dynamics and perturbations in the non-slow roll regime that follows the super-inflation, we showed that the loop quantum effects can indirectly lead to a running of the spectral index, with some loss of power on the largest scales. It should be noted that a similar loss of power on the largest scales can also be obtained via string theoretic scenarios (see Ref. [11] and references therein). It would be interesting to distinguish between the different cosmological effects predicted by loop quantum gravity and string theories, using future high-precision observations.

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