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Covariant Effective Action for Loop Quantum Cosmology à la Palatini

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In loop quantum cosmology, non-perturbative quantum gravity effects lead to the resolution of the big bang singularity by a quantum bounce *without* introducing any new degrees of freedom. Though fundamentally discrete, the theory admits a continuum description in terms of an effective Hamiltonian. Here we provide an algorithm to obtain the corresponding effective action, establishing in this way the covariance of the theory for the first time. This result provides new insights on the continuum properties of the discrete structure of quantum geometry and opens new avenues to extract physical predictions such as those related to gauge invariant cosmological perturbations.

Understanding the nature of gravity and spacetime at high energies is one of the most interesting open issues in theoretical physics. In it lie the answers to various questions which Einstein's theory of general relativity (GR) fails to address, such as the origin of our Universe and the resolution of the big bang singularity. This is also deeply connected with our understanding of the way various dynamical and structural properties of the spacetime and the field equations, such as covariance, emerge from a more fundamental description.

It is generally believed that limitations of GR would be overcome in a quantum theory of gravity, which is expected to cure the big bang singularity and provide modifications to the Friedman dynamics in the early universe. An approach in this direction is to find a renormalizable perturbative theory of quantum gravity which agrees with GR at low energies. This inspired modifications of the Einstein-Hilbert action via addition of terms involving higher curvature invariants and higher derivatives of the metric, motivating ansatzes to potentially tame the initial singularity (see for example [1]). They inevitably have more degrees of freedom than GR and often face limitations such as lack of unitarity, ghosts, and instabilities. These effective theories are based on a classical continuum spacetime and are covariant by construction.

To faithfully capture the dynamical nature of spacetime, however, we need to go beyond the perturbative methods. One such approach is loop quantum gravity, which is background independent and non-perturbative [2]. It is a canonical quantization of gravity with classical phase space given by the Ashtekar variables: the connection A_a^i and the triad E_i^a . A key prediction of the theory is the discreteness of the eigenvalues of geometrical operators such as volume and area. Thus, the classical notion of a smooth differentiable geometry is replaced by a discrete quantum geometry. Techniques of LQG have been successfully applied to formulate loop quantum cosmology (LQC) which is a non-perturbative quantization of cosmological spacetimes [3]. In recent years, extensive analytical work and numerical simulations have shown that the big bang singularity can

be resolved in LQC. The non-perturbative quantum geometric effects result in a quantum bounce to a pre-big bang branch when the energy density of the universe reaches close to the Planck scale [4]. Further, analysis from exactly solvable models show that the bounce is generic [5].

Though the fundamental description in LQC is discrete, it is interesting to note that it admits an effective continuum spacetime description which successfully captures the quantum gravity effects at high energies and becomes classical at low energies. It is derived from an effective Hamiltonian obtained using coherent state techniques. The resulting equations of motion yield trajectories which are in excellent agreement with the quantum expectation values for the states corresponding to realistic universes [6]. As expected, these non-singular trajectories do not exactly follow classical GR but correspond to a modified Friedman dynamics leading to a bounce at the value of the energy density predicted by the quantum theory and recovering classical GR at late times.

These features and success of LQC allow us to pose questions about aspects which were previously poorly understood or unknown. One of such questions is: How do the classical properties of spacetime change when quantum gravitational effects become important? A related question often posed for any canonical quantization is the fate of spacetime covariance. If the fundamental picture is discrete, this issue becomes trickier. Since in LQC an effective continuum description is available, this question can be posed and it is pertinent to ask: Does the effective dynamics of LQC which results in a non-singular evolution correspond to a covariant description? Note that even though, LQC is a quantization only of cosmological spacetimes, it is one of the few settings in 3+1 dimensions where a non-perturbative quantization has been completely performed and physics beyond classical gravity is well understood. Thus, this query holds promise in providing us with a better understanding of at least some of the quantum features of spacetime.

One of the ways to verify if the theory is covariant

is to show that it can be derived from a (covariant) action. Before we find this action for LQC, let us first note one of its interesting features. That is, the effective dynamics of LQC resolves the big bang singularity *without* the introduction of any new degrees of freedom. It means that the corresponding modified Friedman dynamical equations are second order in time, as in GR. This is in contrast with the conventional action based perturbative treatments. Here it is important to recall that requiring second-order equations and covariance one is uniquely led to the Einstein-Hilbert lagrangian density (modulo a cosmological constant) and hence to the Einstein field equations. With these apparent tensions, proving the covariance of the effective dynamics of LQC comes as a challenging task requiring key new insights.

A way out of these problems starts by noting that in formulations with actions involving higher order derivatives of the metric one assumes the compatibility of the spacetime connection with the metric. In LQG, the Ashtekar-Barbero connection is not a spacetime connection [7]. Further, there exists no corresponding connection operator in the quantum theory. It is thus conceivable that the process of loop quantization takes us beyond the compatibility condition between the connection and the metric, changing the description of spacetime in a fundamental way. Hence, when looking for an effective covariant action for LQC, there is no reason to assume any a priori relationship between the spacetime connection and the metric.

If in the gravitational action metric and connection are treated as independent fields one deals with a metric-affine theory. If the connection is torsionless and uncoupled to matter then one arrives at the Palatini formulation of gravity. In this latter approach particles follow geodesics of the Levi-Civita connection of the metric rather than those of the independent connection. Only for the Einstein-Hilbert lagrangian do metric and Palatini formalisms lead to the same dynamics. In general, they are completely different theories. Unlike the metric formulation, where higher order terms motivated from perturbative techniques in quantum gravity have been well studied, so far there is little “inspiration” from a fundamental theory to consider higher curvature actions in the Palatini framework. Phenomenological investigations of Palatini theories have recently gained some attention in relation with the late time cosmic acceleration [8]. Though catastrophic matter instabilities have been found regarding Palatini models with infrared modifications of gravity [9, 10, 11], it is also true that models with ultraviolet modifications are as robust as GR within the experimental limits for suitable choices of model parameters [12, 13].

The most general Palatini action can be represented as a function of the form $f(\mathcal{R}, \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}, \mathcal{R}_{\alpha\beta\gamma\delta}\mathcal{R}^{\alpha\beta\gamma\delta}, \dots)$, where one may include covariant derivatives of functions of the metric and derivatives of curvature invariants.

Here \mathcal{R} denotes the Riemann curvature of the independent connection. In general, the field equations of these theories have the same number of degrees of freedom as GR. This is due to the fact that the independent connection satisfies a constraint equation, rather than a dynamical evolution equation, whose solution can be expressed as the Levi-Civita connection of an auxiliary metric related to the spacetime metric and the energy-momentum tensor of matter. In the simplest case in which the action is of the form $f(\mathcal{R})$, the auxiliary and spacetime metrics are conformally related, with the conformal factor being a function of the trace of the energy-momentum tensor of matter. The connection can then be readily solved in terms of the spacetime metric and the matter and eliminated from the field equations. Though the resulting theory has the same configuration space as GR, its dynamics is different. The role of the Palatini lagrangian $f(\mathcal{R})$ is just to change the way matter generates the spacetime curvature, i.e., it modifies the GR relation $R = -\kappa^2 T$ to arbitrary $f(\mathcal{R})$ lagrangians [see (5) below]. This is to be contrasted with the metric formulation, where the lagrangian $f(R)$ turns the scalar curvature R into a dynamical entity which satisfies a second-order differential equation [14] and, therefore, the theory has higher degrees of freedom.

The kinematical similitudes between LQC and Palatini theories raise a compelling question: Can the effective dynamics of LQC be expressed in the form of an effective Palatini theory? If the answer is positive then apart from establishing covariance of the modified Friedman dynamics in LQC and gaining insights on effects of quantum gravity on the properties of continuum spacetime, a multitude of benefits result. We will be able to understand the way non-perturbative canonical quantum gravity effects can be captured in the effective action treatments. Further, many interesting questions which are beyond the scope or are difficult to address in conventional Hamiltonian treatments could be posed and answered.

Let us consider a flat isotropic and homogeneous FRW spacetime sourced with a massless scalar field ϕ with canonical momentum p_ϕ (satisfying $\{\phi, p_\phi\} = 1$). This model has been successfully quantized in LQC and its physics has been well understood [4]. The underlying quantum constraint is non-local and uniformly discrete in volume. An effective description of the quantum dynamics can be obtained using geometric methods where one treats the Hilbert space as an infinite dimensional quantum phase space with a fiber bundle structure. Using coherent states, an approximately horizontal section which is preserved under the quantum Hamiltonian flow to a desired accuracy can be obtained [6]. The resulting effective Hamiltonian (or the modified Friedman dynamics) describes the underlying quantum evolution extremely well at all scales for universes which grow to a

macroscopic size and leads to a rich phenomenology (see for eg. [15]).

The modified Friedman equation resulting from the effective Hamiltonian methods in LQC (for a massless scalar) is [6]:

$$3H^2 = \kappa^2 \rho \left(1 - \frac{\rho}{\rho_c}\right), \quad \rho_c := \frac{\sqrt{3}}{16\pi^2 \gamma^3 G^2 \hbar} \quad (1)$$

where $\kappa^2 = 8\pi G$. Since, the loop quantization does not affect the matter Hamiltonian, the Klein-Gordon equation and the conservation law are unmodified, i.e. $\dot{\rho} = -3H(\rho + P)$ (where ρ and P are energy density and pressure of the scalar field respectively). It is straightforward to see that when $\rho \ll \rho_c$, the modified Friedman dynamics reduces to the classical equations. At $\rho = \rho_c$, the Hubble rate vanishes and also $\ddot{a} > 0$, implying the bounce of the universe. The occurrence of the bounce is purely a quantum gravitational effect which disappears when $G\hbar$ vanishes (implying divergence of ρ_c).

To obtain the modified Friedman dynamics of LQC from a covariant Palatini action framework, our approach will be to solve the inverse problem – find the Lagrangian, given the equations of motion. Such an action will be effective in the sense that it provides a covariant description of LQC dynamics as obtained from the effective Hamiltonian [6]. A generalized Palatini action is given by

$$S(g, \Gamma, \phi) = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(\mathcal{R}, \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu}, \dots) + S_{\text{matt}}(g_{\mu\nu}, \phi) \quad (2)$$

with $\mathcal{R} := g^{\mu\nu} \mathcal{R}_{\mu\nu}(\Gamma)$. For simplicity we consider the gravitational part only as a function $f(\mathcal{R})$. Its variation with respect to the metric and connection results in

$$f'(\mathcal{R}) \mathcal{R}_{\mu\nu}(\Gamma) - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} = \kappa^2 T_{\mu\nu} \quad (3)$$

$$\nabla_{\mu}^{\Gamma} [\sqrt{-g} f'(\mathcal{R}) g^{\alpha\beta}] = 0 \quad (4)$$

where the prime denotes derivative with respect to \mathcal{R} . The covariant derivative ∇_{μ}^{Γ} is not compatible with the metric: $\nabla_{\mu}^{\Gamma} g_{\alpha\beta} \neq 0$. However, it satisfies $\nabla_{\mu}^{\Gamma} t_{\alpha\beta} = 0$ where $t_{\mu\nu} = f' g_{\mu\nu}$. Thus, Γ is the Levi-Civita connection of the auxiliary metric $t_{\mu\nu}$. The trace of Eq. (3) leads to a generalization of the algebraic relation $R = -\kappa^2 T$ for non-linear $f(\mathcal{R})$ in Palatini:

$$\mathcal{R} f'(\mathcal{R}) - 2f(\mathcal{R}) = \kappa^2 T. \quad (5)$$

This algebraic equation can be solved to obtain $\mathcal{R} = \mathcal{R}(T)$. Inserting the solution for the connection, in terms of $f'(\mathcal{R}(T))$ and $g_{\mu\nu}$, in (3) one finds

$$G_{\mu\nu}(g) = \frac{\kappa^2}{f'} T_{\mu\nu} - \frac{\mathcal{R} f' - f}{2f'} g_{\mu\nu} + \frac{1}{f'} (\nabla_{\mu} \nabla_{\nu} f' - g_{\mu\nu} \square f') - \frac{3}{2f'^2} \left(\partial_{\mu} f' \partial_{\nu} f' - \frac{1}{2} g_{\mu\nu} (\partial f')^2 \right) \quad (6)$$

where $G_{\mu\nu}(g) := R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g)$. Note that Eqs.(3) and (6) are conformally related and the latter implies the former. Further, the conservation law is unmodified, i.e. $\nabla_{\mu} T^{\mu\nu} = 0$. Note also that in vacuum we find $G_{\mu\nu} = -\Lambda g_{\mu\nu}$, with $\Lambda = \frac{\mathcal{R} f' - f}{2f'}$ evaluated at $T = 0$, which recovers the dynamics of GR plus a cosmological constant. This guarantees that the Cauchy problem in vacuum is well-posed (the opposite, however, has been claimed in [16]). From (6), the modified Friedman equation becomes

$$3H^2 = \frac{f'[2\kappa^2 \rho + \mathcal{R} f' - f]}{2 \left(f' + \frac{f''}{2} \frac{\dot{\mathcal{R}}}{H} \right)^2} \quad (7)$$

where $\dot{\mathcal{R}}/H = -12 \kappa^2 \rho / (\mathcal{R} f'' - f')$. It is to be emphasized that since $f(\mathcal{R})$ is a function of T , the right hand side of (7) does not involve any higher derivatives of geometrical quantities and is just a function of the matter sources. The problem of finding the effective action for the LQC dynamics thus reduces to finding an $f(\mathcal{R})$ satisfying

$$\frac{f'[2\kappa^2 \rho + \mathcal{R} f' - f]}{2 \left(f' + \frac{f''}{2} \frac{\dot{\mathcal{R}}}{H} \right)^2} = \kappa^2 \rho \left(1 - \frac{\rho}{\rho_c}\right) \quad (8)$$

where $\rho = \rho(\mathcal{R})$ is a solution to (5). This is just a second-order differential equation for $f(\mathcal{R})$

$$f'' = -f' \left(\frac{f' A - B}{2(\mathcal{R} f' - 3f) A + \mathcal{R} B} \right) \quad (9)$$

where $A = [2(\mathcal{R} f' - 2f)(2\mathcal{R}_c - (\mathcal{R} f' - 2f))]^{1/2}$ and $B = 2[\mathcal{R}_c f' (2\mathcal{R} f' - 3f)]^{1/2}$ and $\mathcal{R}_c \equiv \kappa^2 \rho_c$. Physically acceptable solutions should be free of singularities and have the property that the function $\rho(\mathcal{R})$ maps the full range of values $\rho \in [0, \rho_c]$. These conditions are equivalent to demanding that the acceleration \ddot{a} at the bounce, $\dot{a} = 0$, be the same for both LQC and Palatini $f(R)$, which implies that the bounce must occur at $R = -12\mathcal{R}_c$, where $f' \rightarrow 0$ and $\rho \rightarrow \rho_c$. Numerically we find a family of solutions which converge to a unique function satisfying the above constraints. This shows that *a physically consistent $f(\mathcal{R})$ solution to (9) corresponding to the effective dynamics of LQC exists*. Furthermore, the inverse problem has a unique solution. It is important to note that at curvatures $|\mathcal{R}| \ll \mathcal{R}_c$, $f'(\mathcal{R})$ approximates unity, i.e., $f(\mathcal{R}) \approx \mathcal{R}$, and the solution leads to the classical Friedman dynamics.

An analytical form for $f(\mathcal{R})$ can be obtained by means of interpolation techniques. At low curvatures the numerical solution can be approximated via

$$f(\mathcal{R}) = - \int d\mathcal{R} \tanh \left(\frac{5}{103} \ln \left[\left(\frac{\mathcal{R}}{12\mathcal{R}_c} \right)^2 \right] \right). \quad (10)$$

An interesting function which captures the loop quantum dynamics from sufficiently low to the maximum value of

the curvature is

$$f(\mathcal{R}) = \frac{\mathcal{R}}{12} \left(1 - \frac{1}{2} \ln \left[\left(\frac{\mathcal{R}}{12\mathcal{R}_c} \right)^2 \right] \right) + \frac{\mathcal{R}(\mathcal{R} + 12\mathcal{R}_c)^2}{6500\mathcal{R}_c^2}. \quad (11)$$

The first term dominates at higher curvatures and incorporates the non-perturbative quantum gravity effects that lead to the cosmic bounce in LQC when $\rho = \rho_c$ (see Fig. 1). Note that in contrast with the conventional metric formulation, which generally incorporates perturbative quantum gravity effects via a finite number of terms, the above analytical forms are infinite series. This distinction primarily arises because in non-perturbative LQC quantum geometry effects are non-local. If the fundamental theory were local, a finite number of terms would have sufficed.

Our investigation to find a covariant action for the effective dynamics of LQC provides a much needed motivation from a fundamental description to study $f(\mathcal{R})$ modifications of gravity and its possible extensions in the Palatini formalism. The covariant action we find here leads to non-singular isotropic cosmological dynamics mimicking that of LQC. Based on this action, generalizations to other cosmologies and black hole spacetimes can be considered, which opens a rich avenue to study non-singular spacetimes in the Palatini approach. Further, the availability of an action framework opens a straightforward way to perform a gauge invariant investigation of cosmological perturbations in loop cosmology which is very important to extract physical predictions from the theory.

Let us now revisit the primary question we posed in this letter, namely, the way properties of classical spacetime are affected in a quantum theory of gravity. We considered here as an example non-perturbative loop quantization of cosmological spacetimes. The underlying geometry at the fundamental level is discrete, however the theory admits an effective continuum spacetime. The resulting dynamics, though non-singular, was never established to be covariant until now. Demanding its covariance, we find that the connection must be regarded as independent of the metric in the derivation of the field equations. Though this procedure might not be the only solution to the problem considered here, it provides new insights on the kind of fields that an action should contain to capture non-perturbative quantum gravity effects. This suggests that, unlike in the classical spacetime of GR, the metric might not be the sole fundamental geometric entity. The role of the independent connection is, however, unconventional in the sense that it satisfies a constraint equation rather than a dynamical second-order, differential equation. This, in turn, is what guarantees the kinematical similarities between LQC and Palatini theories. The solution to the constraint shows that matter and geometry get entangled in a non-trivial way with important consequences. In fact, it turns out that the spacetime metric

depends on the local energy-momentum densities, which leads to strong (and unacceptable) backreaction effects in infrared-corrected models [12] but removes the big bang singularity in models with appropriate ultraviolet corrections, as shown here.

Though surprising at first, departures from a purely metric-based framework have been often considered as a necessary requirement if we wish to overcome the limitations of GR, such as non-renormalizability [17]. It has also been argued that if the fundamental description is discrete, like in the crystalline structure of solids, then the metric alone is insufficient to capture all the geometric properties and the effective continuum spacetime may be non-Riemannian [18]. As in a Bravais lattice, the underlying structure in LQC is discrete and our results show that its effective continuum spacetime indeed takes us beyond metric properties. This holds similarity with investigations on studies of continuum properties of crystals [19].

To summarize, we have shown for the first time that despite apparent tensions with the conventional wisdom based on perturbative ideas (in metric formalism), a covariant effective action which reproduces the dynamics of LQC exists in a framework in which metric and connection are regarded as independent.

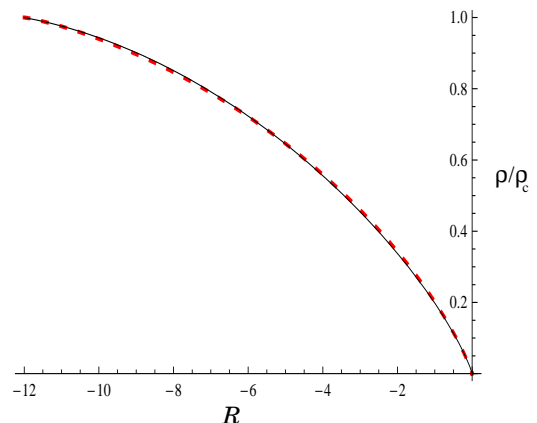


FIG. 1: The energy density in units of ρ_c is plotted for the numerical solution (dashed line) and the $f(\mathcal{R})$ in (11) (solid line). At $\mathcal{R} = -12\mathcal{R}_c$, both of them yield $\rho = \rho_c$ leading to a non-singular bounce.

Loop quantization, at least for the simplest cosmological models, seems to take us away from metric-connection compatibility, thus allowing reconciliation between the lack of new degrees of freedom in the modified Friedmann dynamics and covariance. It remains to be seen whether these novel features survive a more general quantization. Further improvements in the approximations used in obtaining the effective Hamiltonian of LQC [6] and its generalization to include corrections such as those originating from quantum properties of state and to the anisotropic and inhomogeneous spacetime would result

in further insights on the continuum properties of the discrete structure of quantum geometry. Our analysis should be seen as a first step towards such explorations, which provide a glimpse of new ways in which matter and geometry might get entangled, via an independent connection, and the nature of the effective spacetime emergent from a quantum geometry.

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