10-1-2003

Discrete quantum gravity: A mechanism for selecting the value of fundamental constants

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Discrete quantum gravity: a mechanism for selecting the value of fundamental constants *

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(June 20th 2003)

Smolin has put forward the proposal that the universe fine tunes the values of its physical constants through a Darwinian selection process. Every time a black hole forms, a new universe is developed inside it that has different values for its physical constants from the ones in its progenitor. The most likely universe is the one which maximizes the number of black holes. Here we present a concrete quantum gravity calculation based on a recently proposed consistent discretization of the Einstein equations that shows that fundamental physical constants change in a random fashion when tunneling through a singularity.

Fundamental constants in nature need to fall within a rather narrow set of values for the universe to have its current form, in particular to accommodate life. When written in dimensionless form, unnaturally large ratios appear between various of the fundamental constants. Inflation has been proposed as a mechanism to account for several features of the universe, but it cannot explain the values of all fundamental physical constants nor provide a complete resolution to the hierarchy problem. A recent proposal due to Smolin [2] poses that the selection of the values of the fundamental physical constants happens through a Darwinian process. Whenever a universe develops a black hole a new universe forms within it with different values of the physical constants. The universes that are naturally selected are those such that the physical constants are such that they maximize the likelihood of formation of black holes. That allows such universes to reproduce more efficiently. This attractive proposal has the feature that it can be tested experimentally. It can be falsified by showing that the values of the physical constants are such that we are not at the maximum likelihood of formation of black holes. The proposal has been recently reconsidered by Bjorken [3]. Several criticisms have been levied against these arguments. One of the problems up to now has been the lack of a detailed mechanism to account for the change of fundamental physical constants during the tunneling through a black hole.

In this paper we would like to discuss a detailed scenario in which changes in the fundamental physical constants can occur when tunneling through a singularity. Having a detailed scenario for tunneling might be interesting cosmologically even independently from Smolin's Darwinian hypothesis [4].

The proposed scenario is based on the recently introduced consistent discretization technique for treating quantum general relativity on the lattice [1]. The technique constructs a discrete theory on the lattice that represents an approximation to general relativity and such that all of its equations can be solved simultaneously (usual discretizations of general relativity produce an inconsistent set of equations). The discrete theories constructed with the new technique have several attractive features. Among them is the presence of well understood symmetries that provide a lattice representation of the symmetries of general relativity [5]. This is quite novel, since it has been a long standing problem how to reconcile the continuous coordinate invariance of general relativity with the discreteness of a lattice framework.

In this letter we analyze a concrete example of a bounce through a singularity in the consistent lattice approach. We will consider a Friedman universe with a cosmological constant and a (very massive) scalar field. This is the simplest model we have found that exhibits bounce through a singularity. As discussed in [5], anisotropic models exhibit similar behavior. The approach to the singularity in the interior of a black hole can be modeled as an anisotropic cosmology and therefore the following discussion is of relevance to the behavior of the interior of a black hole. It should be noticed that several mechanisms have been postulated in the past for tunneling through a black hole. Some of these have been classical, postulating the development of a cosmological constant in the interior [6], quantum inspired modifications of general relativity [4], or path integral formulations of quantum gravity [7], but most of these have not been associated with changes in the fundamental physical constants, although see [8].

The Lagrangian for the model, written in terms of Ashtekar's variables [9] is,

$$L = E\dot{A} + \pi\dot{\phi} - NE^2(-A^2 + (\Lambda + m^2\phi^2)|E|) \tag{1}$$

where Λ is the cosmological constant, m is the mass of the scalar field ϕ, π is its canonically conjugate momentum and N is the lapse with density weight minus one. The appearance of $|E|$ in the Lagrangian is due to the fact that

*This essay received an "honorable mention" in the 2003 Essay Competition of the Gravity Research Foundation – Ed.

the term cubic in E is supposed to represent the spatial volume and therefore should be positive definite. In terms of the ordinary lapse α we have $\alpha = N|E|^{3/2}$.

We consider the evolution parameter to be a discrete variable. Then the Lagrangian becomes

$$L(n, n+1) = E_n(A_{n+1} - A_n) + \pi_n(\phi_{n+1} - \phi_n) - N_n E_n^2(-A_n^2 + (\Lambda + m^2\phi_n^2)|E_n|) \tag{2}$$

The discrete time evolution is generated by a canonical transformation of type 1 whose generating function is given by $-L$, viewed as a function of the configuration variables at instants n and $n+1$. Evolution equations can be written for all the variables and their canonical momenta. The evolution equations are made consistent by determining the Lagrange multipliers N_n. The resulting equations can be reduced to [5],

$$P^A_{n+1} = A_n^2 \Theta^{-1} \tag{3}$$

$$A_{n+1} = \frac{3A_n^2 - P_n^A \Theta}{2A_n} \tag{4}$$

$$\phi_{n+1} = \phi_n \tag{5}$$

$$P^\phi_{n+1} = P^\phi_n - \left(A_n^3 - P_n^A \Theta A_n\right) m^2 \phi_n \Theta^{-2} \tag{6}$$

where $\Theta = \Lambda + m^2\phi_n^2$. It should be noted that these equations preserve the symplectic structure, that is, the variables (P^A_{n+1}, A_{n+1}) have the same canonical Poisson brackets as (P^A_n, A_n). To make contact with the variables of the continuum, we note that the triad $E_n = P^A_{n+1}$.

The discrete evolution equations have the feature that they avoid the singularity present in the continuum model for generic sets of initial data [10]. In figure 1 we show a generic evolution near the region where classically one would encounter a singularity. One can see that the discrete theory, although approximating reasonably well the continuum behavior does not have the metric going through zero.

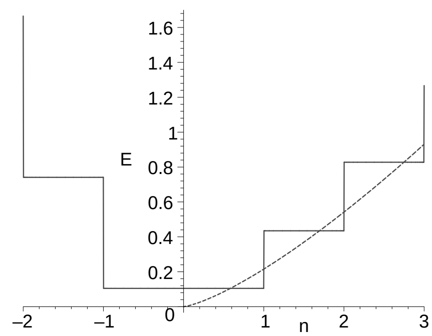

FIG. 1. The approach to the singularity in the discrete and continuum solutions. The discrete theory has a small but non-vanishing triad at $n = 0$ and the singularity is therefore avoided.

It should be noted that the resulting theory has no constraints, unlike the continuum theory [11]. Therefore one does not confront the problem of finding "observables", that is, quantities that have vanishing Poisson brackets with the constraints. The discrete theory has four phase space degrees of freedom and one can introduce four constants of motion. Three of these constants of motion do not depend explicitly on the evolution parameter n. Remarkably, two of them can be viewed as discretizations of the two independent observables of the continuum theory. Therefore the discrete theory has in a precise sense embedded in it the symmetries of the continuum theory [5]. The symmetries are present in the discrete theory in the sense that there exist constants of the motion independent of the evolution parameter that one can use to generate canonical transformations representing the symmetries.

The remaining two constants of motion vanish in the continuum limit. Let us concentrate on the one that is independent of the evolution parameter. It arises from considering the canonical transformation that generates time evolution as an exponentiation of a quantity that plays a role of generalized Hamiltonian for the discrete model (the model does not have a genuine Hamiltonian since time is discrete). The generalized Hamiltonian should therefore be preserved under evolution. If one recasts the evolution equation as,

$$A_{n+1} = A_n + \{A_n, H_n\} + \frac{1}{2!}\{\{A_n, H_n\}, H_n\} + \ldots \tag{7}$$

and similarly for the other variables, one can read off the "Hamiltonian"

$$H_n = \frac{\mathcal{C}_n^2}{4\Theta A_n}\left[1+\sum_{k=1}^{\infty} a_k \left(\frac{\mathcal{C}_n}{A_n^2}\right)^k\right] \tag{8}$$

where $\mathcal{C}_n = \left(A_n^2 - P_n^A\Theta\right)$ is the discretization of the Hamiltonian constraint of the continuum theory and $a_1 = 1/(3\times 4), a_2 = 1/(6\times 4^2), a_3 = 0, a_4 = -1/(6\times 4^4), a_5 = -1/(15\times 4^5), a_6 = 7/(10\times 4^6)$ etc. The power series nature of the definition of this constant implies that it exists only where the series is convergent. Whenever it is, the finite canonical transformation can be written as an exponentiation of an infinitesimal canonical transformation (contact transformation). To understand this, notice that the canonical transformation that materializes the discrete evolution is singular when $A_n = 0$ (see equation (4). This singularity separates the phase space into two disjoint regions $A_n > 0$ and $A_n < 0$. The contact transformation will fail to exist when the canonical transformation connects points that lie different disjoint regions, i.e. when $\mathrm{sg}(A_{n+1}) \neq \mathrm{sg}(A_n)$. This happens when $3A_n^2 - P_n^A\Theta < 0$. This in turn implies that the expansion parameter of the series ("normalized constraint") $|\mathcal{C}_n/A_n^2| > 2$. The prediction is therefore that this will be a conserved quantity until the evolution takes us over the singularity in the canonical transformation.

As seen in figure 2 the rate of expansion/contraction changes when tunneling through the singularity. The constant of the motion can be seen as an invariant characterization of such a rate. More precisely, the constant of the motion, which vanishes in the continuum limit, is a measure of how well the discrete theory is approximating the continuum one. What changes in the tunneling is the lattice spacing.

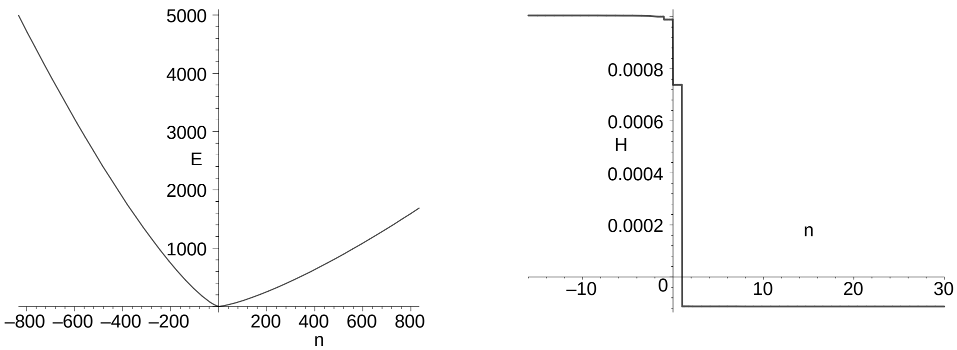

FIG. 2. Typical behavior of the discrete evolution of the triad E and the constant of motion H. One can see that the rate of expansion/contraction differs when tunneling through the singularity. This is reflected itself in a jump in the value of the constant of the motion H.

It is worthwhile noticing that the magnitude of the jump in the constant of the motion exhibits sensitive dependence on the initial condition of the problem. A small change in the initial values can lead to large changes in the behavior of the jump. To view it in another words, the rate of expansion after the tunneling loses correlation with respect to the rate of contraction before the tunneling. This can be seen in figure 3.

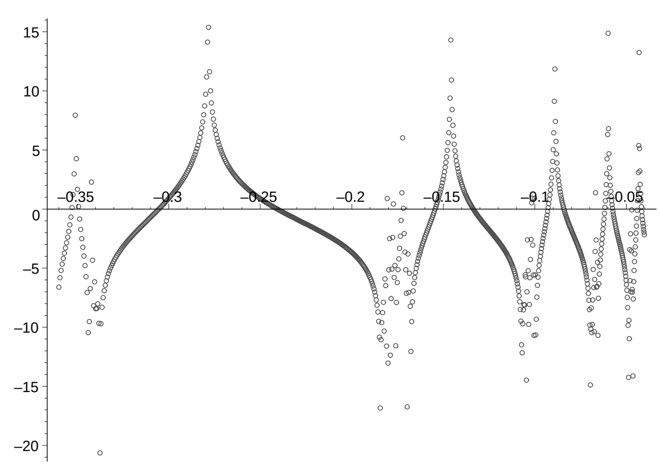

FIG. 3. The value of the logarithm of the observable after the bounce as a function of the initial value of the observable. We see that there exist ranges in which very small variations of the initial value translate themselves in large changes in the final value. This shows that the change in the fundamental constants in black hole tunneling is not deterministic, leading to a Darwinian picture.

At this point the reader may wonder what is the connection between the calculation we presented of the "bounce" in the value of an observable in a model cosmology and the values of the fundamental physical constants. Of course, a detailed model involving interacting fields and local degrees of freedom will require a much larger calculational effort than what we are able to attempt at present. At this point, we can only present a heuristic argument. The argument is based on the fact that the observable we discussed is an invariant measure of the lattice separation. Therefore its value is connected with how refined the lattice spacing in the theory is. In this particular model, the lattice is only in the time-like direction, but one can expect that in more realistic models, with local degrees of freedom, similar behaviors will occur for the spatial lattice spacings. Now, in a lattice gauge theory the values of the fundamental physical constants are related to the bare values that appear in the Lagrangian through a limiting process in which one takes the lattice spacing to zero and also fine-tunes the bare parameters in such a way that the "dressed" physical constants are finite. More precisely, such a process is fine tuned for one observable, and then the same process predicts the values of other observables (at least for renormalizable theories). In the gravitational case we do not expect to have renormalizability in the traditional sense, so the proposal we are presenting is that the theory remains discrete. In the discrete theory there will exist states that approximate the continuum theory better than others and a measure of this will be given by the value of the observable we discuss. The value of the "dressed" physical constants will depend therefore on the bare values and on the value of the "spacing". An invariant measure of the "spacing" in the lattice is given by the value of the "Hamiltonian" (8). In a situation with "fine" spacing its value will be small. In this scenario, the tunneling through the singularity we have exhibited will translate itself in a change in the value of the "Hamiltonian" and therefore in a change in the value of the fundamental constants.

The discussion up to now has been entirely classical. However, the discontinuity in the constant of the motion has a quantum counterpart. To quantize the system, one represents the quantum evolution via a unitary transformation that implements at the level of Heisenberg equations the discrete equations of motion associated with the canonical transformation we discussed above. We will not give the details here, some of them can be seen in [5]. In the quantum theory, one can consider a state peaked around a classical solution and evolve it using the discrete evolution operator. One cannot directly promote the "Hamiltonian" to a quantum self-adjoint operator since it is not well defined near the bounce. However, one can take a finite number of terms of its expansion. This will approximate well (classically) the behavior of the "Hamiltonian" far away from the bounce. Such a finite expansion can be promoted to a self-adjoint quantum operator. This allows, for instance, to compute its expectation value before and after the point where one would have expected the big bang classically. Generically the values will be different, mirroring what we found in the classical theory.

The singularity that arises in the big bang has elements in common with the singularity in the interior of black holes. Although the cosmological model we studied in detail is not the one that has elements in common with the black hole interior, the feature we presented of tunneling through the singularity is expected to exist in a variety of models. Also, the details of how the changes occur could differ with different discretization schemes. We can therefore expect a similar phenomenon to be present in the interior of black holes, although the details may vary. Each black hole will have its singularity replaced by tunneling into a new universe, in which the dressed value of the fundamental constants will be different. The change in the values has elements of randomness in it. This allows to construct a picture of the universe in which "evolution" takes place every time a black hole is formed, as was the original proposal of "The life of the cosmos" [2].

We wish to thank D. Ahluwalia for comments. This work was supported by grants nsf-phy0090091, funds of the Horace Hearne Jr. Institute for Theoretical Physics, the Fulbright Commission in Montevideo and PEDECIBA (Uruguay).

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