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Loss of quantum coherence from discrete quantum gravity

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We show that a recent proposal for the quantization of gravity based on discrete space-time implies a modification of standard quantum mechanics that naturally leads to a loss of coherence in quantum states of the type discussed by Milburn. The proposal overcomes the energy conservation problem of previously proposed decoherence mechanisms stemming from quantum gravity. Mesoscopic quantum systems (as Bose–Einstein condensates) appear as the most promising testing grounds for an experimental verification of the mechanism.

It is commonly believed that a satisfactory theory of quantum gravity may require a drastic modification of our description of space-time. Among the proposed fundamental changes that appear in the literature is that the ultimate theory may imply a discrete structure for space-time at a microscopic level. Recently, a quantization of gravity in discrete space-time has been developed which addresses major fundamental concerns of the canonical program [1]. Among the appealing elements of the proposal is the solution of the problem of time in generally covariant systems through the introduction of relational time in quantum mechanics [2]. Having promoted time into the quantum realm it is meaningful to ask how to compare the resulting theory with the traditional Schrödinger picture. We shall show in what follows that in the semiclassical limit, discrete quantum gravity may lead to information loss in quantum states. We will also argue that the process can potentially lead to observable consequences. This construction leads to the same density matrix evolution equation that has been considered by several authors [3, 4, 5, 6, 7] in other contexts. It can also be viewed as a concrete implementation of the idea of Penrose [8] that gravity should play a role in the state reduction process. The equation is given by,

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] - \mathcal{D}(\rho), \quad (1)$$

with $\mathcal{D}(\rho)$ a decoherence term which has been usually taken as having the modified Lindblad's form[3],

$$\mathcal{D}(\rho) = \sum_n [D_n, [\rho, D_n^\dagger]], \quad D_n = D_n^\dagger, \quad [D_n, H] = 0, \quad (2)$$

so it defines a completely positive map on ρ , and which is consistent with the monotonous increase of Von Neumann entropy $S = -\text{Tr}(\rho \log \rho)$ and conservation of energy. This type of equation was introduced by Ghirardi, Rimini and Weber (GRW) [4] with the aim of providing an objective solution to the measurement problem in standard quantum mechanics. (Similar equations can be used to describe the decoherence due to interaction with an environment, see [9].) GRW considered a single D as

a localizing operator in coordinate space. As discussed by Adler and Horwitz [7], and also Milburn, Percival and Hughston [5, 6], setting D to be proportional to H is most natural since it leads to an objective state vector reduction in the energy pointer basis. This loss of coherence may be a way to avoid macroscopic superpositions, like the “Schrödinger cat” [5, 7]. We shall show that starting from the recent proposal for “consistent discrete quantum gravity” of [1] one obtains proportionality between D and H . There are other type of constructions where \mathcal{D} represents collectively quantum gravity effects. In general it does not take the commutator form of equation (2) and it is mainly associated with partial traces over “environmental” degrees of freedom like microscopical black holes or strings [10, 11, 12]. As shown by Hawking [13], these generalized equations may lead to violations of unitarity during black hole evaporation. Hawking's proposal was criticized on the grounds that it violates energy conservation [12]. To find an explicit description of information loss in quantum gravity consistent with energy conservation is one of the major challenges of the field. We will show that the “consistent discrete approach” [1] ensures energy conservation. Therefore one can conjecture that this description may provide a concrete theoretical solution to the black hole entropy problem based on a quantum description of space-time.

Let us now show how discrete quantum gravity leads to $D \propto H$. There have been several attempts to formulate a purely relational description of quantum mechanics [14, 15]. The idea is to consider a system that is closed and try to define a notion of time through the evolution of a sub-system of the closed system. This is of interest conceptually in ordinary quantum mechanics, since it frees the description of its reliance on an external classical clock and is unavoidable when one is considering quantum cosmology, where there simply does not exist an external observer. Perhaps the most explicit examples of attempting to present a detailed description at the quantum mechanical level of a purely relational evolution are the series of papers by Page, Wootters and others [15]. A notion of time is introduced via conditional probabilities, that is, asking what is the probability

that a certain quantity has a certain value when another quantity has a given value. The latter could be viewed as a “quantum mechanical clock” introducing a purely quantum mechanical notion of time. The resulting description is an extension of ordinary quantum mechanics that allows to make predictions in regimes where the notion of a classical clock is not applicable. As discussed by Kuchař [16], there are problems with the relational approach, when applied to totally constrained systems like general relativity due to the presence of constraints. The proposed consistent discrete framework for gravity is constraint-free, and therefore we have been able to show that it is possible to introduce a relational description of time avoiding the hard issues that plagued the previous attempts [2].

Let us consider a concrete application of the consistent discrete approach to a Friedmann cosmology coupled to a scalar field. We will denote the latter by ϕ . Details of a similar model can be seen in [17]. In a Friedmann cosmology set up as a canonical system, the fundamental variable is the spatial metric, which has only one independent component and its canonically conjugate momentum. It is best to describe the model using Ashtekar variables [18]. In these variables the metric is replaced by a triad, which also has only one independent component in the Friedmann case. We denote it by E . Its canonically conjugate momentum is denoted by A . The scalar field and their canonical momenta are (ϕ, P^ϕ) . One writes the Einstein action for the model and discretizes the evolution parameter (since the model is homogeneous) as $\tau = n\Delta\tau$. Evolution can be represented at the canonical level via a canonical transformation that implements the discrete equations of motion from the level n to level $n + 1$. One of the equations (the one that would correspond in the discrete theory to the single constraint of the continuum theory) determines the lapse and is solved for it. The resulting theory therefore has no constraints. All its variables are therefore candidates for physical observables and one is therefore ready for the application of the relational time formalism. The quantization of the theory is given in terms of quantum states $\Psi(A, P^\phi, n)$. The evolution of the theory is unitary in terms of the n variable, since we can represent the canonical transformation as a unitary evolution operator in the quantum theory. We can now use for instance the variable A as a time variable and compute the conditional probability that the scalar field momentum have a certain value at a given “time” as (one should really phrase it in terms of intervals since the variables have continuous spectrum)

$$P_{\text{cond}}(P^\phi = x | A = t) = \lim_{N \rightarrow \infty} \frac{\sum_{n=0}^N \Psi^2[A=t, P^\phi=x, n]}{\sum_{n=0}^N \int_{-\infty}^{\infty} \Psi^2[A=t, P^\phi, n] dP^\phi} \quad (3)$$

In the discrete approach the cosmology appears as a succession of “snapshots” labeled by the integer n , which

lacks any intrinsic meaning. The emergence of time in the model is only through the correlations of the dynamical variables of the theory. The quantum theory that results from the relational probabilities only agrees with ordinary quantum mechanics in regimes in which a notion of classical time is a good approximation to the behavior of a quantum variable [2]. It is clear that generically there could be departures from this regime. Let us denote by ρ the initial density matrix for the gravity-matter system and assume that in the semiclassical limit we can decouple the system as $\rho \approx \rho_1 \otimes \rho_2$, with ρ_1, ρ_2 , associated to geometry and the field respectively. We sketch a proof that there exists a relational Schrödinger picture where there is an effective density matrix for the “system” which evolves in “internal clock” time into a statistical mixture even if it was in a pure initial state with a resulting evolution equation of the form (1,2) with D proportional to H .

We discuss the derivation for a generic system, to particularize it to the cosmology we mentioned before one chooses one of the variables as clock, for instance $t = A$ and $x = P^\phi$. We proceed in two steps. First we introduce the relational Schrödinger picture that our approach follows. We compute the evolution of the density matrix for the “rest” system by summing over all configurations in the variable n that are compatible with a certain time t , $\rho_2(t) \equiv \sum_n \mathcal{P}_n(t) U_2(n) \rho_2(0) U_2^\dagger(n)$, where $\mathcal{P}_n(t) = \text{Tr}[P_t(0) U_1(n) \rho_1 U_1(n)]$. Where $U_1(n), U_2(n)$ are the unitary discrete evolution operators in terms of the discrete parameter n for the “time” and “the rest” variables respectively. We also denote as $P_t(0), P_x(0)$ the projectors in the $n = 0$ level into the sub-space corresponding to the values t, x of the “time” and the field. We will assume now that there exists a Hamiltonian operator for U_2 such that $U_2(n) = \exp(-iH_2n)$. It is important to notice that this evolution differs from the usual Schrödinger picture due to the presence of the sum, which is related to the fact that there is not a unique correspondence between the discrete parameter n and a given value of time t . Therefore the trace of the square of the evolved density matrix will not be one and therefore a pure state evolves into a mixed state. For reasons of space, we do not present the detailed derivation that this evolution equation implies the equations (1,2), but we give the following intuitive explanation. If the probability distribution $\mathcal{P}_n(t)$ were a Dirac delta (one step n is associated uniquely to some time t) then the density matrix would satisfy equation (1) with $\mathcal{D} = 0$. In practice, the probability distribution will be peaked around some value of n with contributions from neighboring values. In the sum this implies that there will be terms representing the evolution from those neighboring values of n . This evolution can be viewed as generated by the action of a Hamiltonian operator. This additional operatorial action is what leads to the double commutator in (2). This can be worked out in detail, we will present the calculation

elsewhere.

Let us make, however, some comments. Consider $t_{max}(n)$, that is, the value of the maximum probability for the variable t as a function of n . We will assume we chose the temporal variable in such a way that $t_{max}(n) = \gamma n$ with γ a constant of the motion and we will denote by $\delta\gamma$ its quantum fluctuations. It is possible to show that $D = \sqrt{\sigma}H$ with $\gamma\sigma = \partial(\delta t_{max})^2/\partial n$, where $\delta t_{max} = (\delta\gamma)n$. What is happening here is that one chose a wavepacket in which t was a peaked function of n for the clock, but as the system evolves such wavepacket spreads out and σ will be a measure of that spread. σ is related to the rate of growth of the spread of the packet.

Equations (1,2) imply that coherence is lost since the off diagonal terms of the density matrix go to zero,

$$\rho_{2nm}(t) = \rho_{2nm}(0)e^{-i\omega_{nm}t}e^{(-\sigma(\omega_{nm})^2)t} \quad (4)$$

where $\omega_{mn} = E_m - E_n$ are the Bohr frequencies (for a derivation see [19], where this formula was obtained by studying a classical non-ideal clock). Notice the loss of coherence implied by the exponential. This can have remarkable effects, for instance if one waits long enough all off-diagonal elements of the density matrix vanish. In spite of this, it is difficult to find experimental situations where these effects are measurable. For instance one may consider light that propagates from distant stars in order to have long times of flight and enhance the effect. However, since most optical measurements imply measuring second (or, more generally, even)-order correlations [20], this loss of coherence has no visible consequences in optics.

In order to seek for possible experiments, let us first estimate the value of σ , which we expect is going to be a small time scale, of the order of the Planck one. We shall begin by using the whole universe as the quantum clock in order to get some intuition to the bounds for these effects to happen. Since the universe is the biggest reservoir we have it is naturally to believe that it is the best clock we can build. Let us assume that our present universe may be modeled by the Friedmann cosmology we discussed before. As it is shown in [1] the discrete evolution for the connection A goes as $l_p\Lambda^{1/2}a(n+k)^{2/3}$ and $E = l_p^2a^2(n+k)^{4/3}$, where $a > 0, k$ are two non-dimensional constants which parameterize the set of equivalent orbits in the continuum limit. Let us now take the relational time as $t = \Lambda^{-3/4}l_p^{-1/2}A^{3/2}$, such that it has dimension of time, and time is thus measured in cosmological units (we also have $\hbar = c = 1$). We have $t = l_p a^{3/2}(n+k)$ which is of the form $t = \gamma(n+k)$, with $\gamma = l_p a^{3/2}$, and linearly in n as we wanted. On the other hand, due the uncertainty principle for E, A , i.e. $\Delta E \Delta A > l_p^2$, we have $a\delta a^2 > \frac{1}{l_p\Lambda^{1/2}(n+k)}$. From this one can immediately estimate a lower bound for $\delta\gamma$ and therefore a lower bound for $\sigma > l_p/(t\sqrt{\Lambda})$. For the present epoch of the universe one therefore has

$\sigma > l_p$. These estimations for σ should not be seen as a concrete calculation. In order to refine them we need to construct a more realistic model of the universe including more complexity. See [21] for a lengthier discussion.

As was first discussed by Ellis et al. [10] there are several possible phenomenological implications of this non standard quantum behavior. Examples are neutron interferometry and the neutral kaon decay. It is easy to see that our approach implies extremely small corrections to the usual quantum mechanical predictions for these systems. All these models suffer from the same problem, they involve small energy differences between channels so our predictions, though of theoretical interest, are extremely small. This contrasts with the string theory predictions considered in [10] where the size of the effect is controlled by the external string background and its energy scale is conjectured to be some exponent of the Planck Mass [12].

A system which has been considered in connection with Milburn's [5] type of decoherence is a two level atom interacting with the electromagnetic field in a cavity [22]. We follow closely the analysis of reference [22]. The system is described by the Hamiltonian $H = H_a + H_f + H_i$ where $H_a = \frac{\omega}{2}R_3$ describes the energy splitting of the two level atom. $H_f = \omega a^\dagger a$ is the traditional number operator for the ω -mode of the field inside the cavity, and $H_i = \lambda(R^+ a + a^\dagger R^-)$ with $\lambda^2 \sim \mu^2\omega$, the dipole coupling constant. Here μ is the dipole matrix element between both levels of the atom. The operators R_3, R^\pm are essentially the traditional angular momentum operators and $(R_3)^2 = 1$. Notice that this model could be used to describe any two level system coupled to a mode of a field which induces transitions. Indeed the population of the upper level is given by $1/2(1 + \langle R_3 \rangle)$. It is easy to see now that within our approach the atomic inversion evolves as,

$$\langle R_3 \rangle_{\sigma} = \sum_s |Q_s(0)|^2 e^{(-2(s+1)\sigma\lambda^2 t)} \cos(2\sqrt{s+1}\lambda t) \quad (5)$$

where $Q_s \equiv e^{-|\alpha|^2/2} \frac{\alpha^s}{\sqrt{s!}}$. We have considered initially the field in a coherent state $|\alpha\rangle$ ($\bar{n} = |\alpha|^2$), and the atom in its excited state. This system was first analyzed by Moya et al. [23] who studied the same model within Milburn's proposal [5] which is dynamically identical to our approach plus the identification $\sigma \rightarrow \theta_0$. The physical conclusions are that coherence is destroyed and "revivals" in the coherence are exponentially suppressed. The loss of coherence is likely to act slower than the decoherence due to ambient factors and therefore one expects that actual experiments will be dominated by the ambient decoherence. In fact, modeling the latter by the introduction of a damping constant γ and Langevin noise leads to a solution for the atomic inversion in the under-damping

limit $\gamma \ll \lambda$, of the form [22],

$$\langle R_3 \rangle_\gamma = \frac{1 - 2\bar{n}}{1 + 2\bar{n}} \left(e^{-\gamma \frac{1+2\bar{n}}{1+\bar{n}} t} - 1 \right) + \sum_s |Q_s(0)|^2 e^{-\gamma t} \cos(2\sqrt{s+1}\lambda t) \quad (6)$$

Notice that both effects are similar but leave a different imprint since the exponential decay does not depend on the number of photons n in the ambient decoherence while it does in the one due to loss of coherence. In order for the effect to be visible one would need a high intensity laser, which would face limitations since high intensities will increase the ambient decoherence of the cavity. It should be noticed that the relative importance of the loss of coherence can be enhanced through the increase of the dipole coupling of the field with the two level atom μ . Therefore an optimal experiment should have intense fields, strong coupling of the atom to the field and widely separated energy levels in the atom. At present this appears beyond the state of the experimental art with these types of experiment.

Since the effect depends on the number of photons, this suggests that systems involving mesoscopic quantum states should be well suited for experimentally probing the effect. An example of such systems could be the Bose-Einstein condensation (BEC). Recently Greiner et al [24] studied the ‘‘collapse and revival’’ of matter wave fields in BEC. They showed that the macroscopical wave function undergoes a series of ‘‘collapses and revivals’’ due to the collisions of cold atoms confined to a potential well. We will not repeat the calculation explicitly here for this model for reasons of space, but we have found an effect very similar to that of equation (5) for this system. The main difference is that in the exponent the number of atoms enters quadratically, as opposed to (5) where the number of photons entered linearly. Although the currently considered experiments with BEC involve very few atoms, and again one is very far away from seeing quantum gravitational effects, it is possible that future experiments with more atoms, giving the quadratic dependence on the number could be more promising. To our knowledge these detection of revivals will, in the future, provide the most stringent bounds on quantum gravity inspired decoherence known up to now. In general experiments that test ‘‘Schrödinger cat’’ type situations can all potentially lead to observations of the decoherence we found. For a recent review see [25]. SQUID experiments seem to provide the best bound up to date [26, 27].

Summarizing, we have shown that a recently introduced proposal for quantizing gravity on discrete space-time leads naturally to a quantum mechanics that includes a fundamental decoherence of the Milburn type. Contrary to Hawking’s information loss proposal, ours does not violate energy conservation. Future experimental developments in quantum mesoscopic systems can lead to a confirmation of the process.

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