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Classical and quantum general relativity: a new paradigm

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Abstract

We argue that recent developments in discretizations of classical and quantum gravity imply a new paradigm for doing research in these areas. The paradigm consists in discretizing the theory in such a way that the resulting discrete theory has no constraints. This solves many of the hard conceptual problems of quantum gravity. It also appears as a useful tool in some numerical simulations of interest in classical relativity. We outline some of the salient aspects and results of this new framework.

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It has perhaps not been quite widely realized that quite a significant portion of current research in general relativity (both at a classical and quantum mechanical level) is conducted through discretizations of the theory. In classical general relativity the numerical integration of the Einstein equations involves a large number of researchers and in fact has been cited as one of the national scientific priorities of the US [1]. The binary black hole problem, for instance, has appeared as particularly challenging. On the other hand, in the realm of quantum gravity, the theory is discretized in order to regularize, for instance, the Hamiltonian constraint (in canonical quantum gravity) or the path integral (for instance in the spin foam approach).

In spite of the prevalent role of discretizations in modern gravitation, there has not been a wide appreciation of the —however widely accepted— fact that the resulting discrete theories have significantly different properties than continuum general relativity. For instance, discrete theories have a completely different symmetry structure than the continuum theory (to put it simply, the discretization process breaks diffeomorphism invariance). In fact, when one discretizes a continuum theory one is producing an entirely new theory. The hope is that such a theory will contain among its solutions some that approximate in certain ways the solutions of continuum general relativity. In spite of this hope, this is usually not the case. For instance in numerical relativity it is well known that if one discretizes the equations of motion one gets an inconsistent set of discrete equations. If one takes initial data that satisfies the constraints and evolves them, the constraints will fail to be satisfied upon evolution. In fact, the solution of the discrete equations of motion contain solutions that drift rapidly away from the constraint surface or that grow out of control. This problem is so pervasive that no long term simulations of binary black holes are currently possible in spite of many years of efforts of a large community, and some researches place the blame squarely on the constraint violations [2].

In the realm of quantum gravity, if one discretizes the constraints of canonical quantum gravity in order to regularize them, the resulting constraints fail to close an algebra [3]. Again, this implies the theory being constructed is inconsistent. Taking successive Poisson brackets of the constraints generates an arbitrarily large set of new constraints. In loop quantum gravity this was an obstacle for many years. Although now there exists a subtle limiting procedure [4] that removes this inconsistency upon quantization, the issue of the constraint algebra still seems to raise questions about the resulting quantum theory [5].

We have recently introduced [6, 7] a procedure for discretizing general relativity that yields equations of motion for the theory that are consistent, i.e. they can be solved simultaneously. The approach has been called "consistent discretization". The idea is very simple: it consists in discretizing the action and working out the Lagrangian equations of motion of the discrete action. In the context of unconstrained systems, this idea is known as "variational integrators" [8]. Generically, this immediately guarantees the resulting discrete equations are consistent. Again, generically, the resulting discrete theories do not have constraints, all equations are evolution equations. The resulting discrete theories have been shown to approximate general relativity in a set of situations of increasing complexity. Several initial reservations about these schemes, like the fact that they could yield unstable or complex solutions or that one loses contact with loop quantum gravity have now been shown not to be fundamental obstacles.

What we would like to point out in this essay is that the newly introduced way to discretize general relativity in fact has turned into a new paradigm for studying gravity. In this approach one is not fixing a gauge and nevertheless one is constraint-free and therefore all variables are observables of the theory. This offers a completely new way to analyze problems in classical and quantum gravity. We will now summarize some of the salient features of the new paradigm.

Classical results:

The discrete theories constructed with the "consistent discretization" approach have several unusual features. To begin with, the lapse and shift become dynamical variables that are determined in the equations of motion. This is in line with the fact that the theory has no constraints. That means it has more degrees of freedom than the continuum theory it approximates. These extra degrees of freedom characterize the freedom to choose the Lagrange multipliers (the lapse and shift). Since the lapse is determined dynamically, this implies that the "time-steps" taken by the evolution change over time. When they are small, the discrete theory approximates the continuum theory well. Interestingly, we have observed in experiments with the Gowdy cosmology [9] (see figure) and simple mechanical systems that solutions can sometime depart from the continuum for a while and later return to approximate the continuum theory very well. This may be related to the fact that the resulting discretization schemes are implicit. It is a feature that can be extremely attractive —if generic— for long term simulations of space-times like the ones that are currently sought in the binary black hole problem. It is also interesting that the schemes are convergent even though no attempt has been made to incorporate ideas of hyperbolicity (this is challenging since most hyperbolic formulations of the Einstein equations that are known are not derivable from an action).

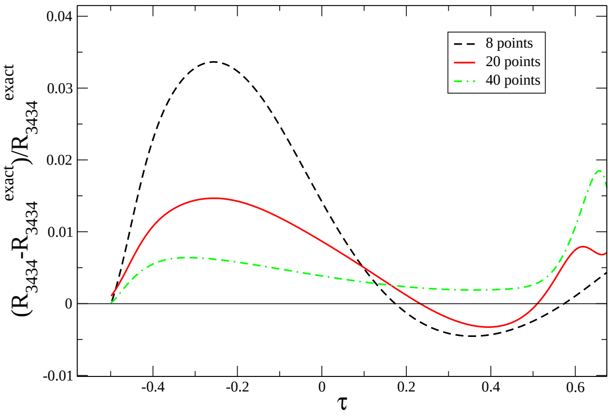

FIG. 1: The value of the Relative error with respect to the exact solution of the 3434 component of the Riemann tensor for a Gowdy cosmology evolved with the consistent discretization scheme as a function of time for $\theta = \pi$. We show three resolutions, 8, 20 and 40 spatial points and one sees that the scheme converges. The errors grow but then, remarkably decrease again. If generic, this feature of recurring back to low error solutions could be a key element for long term simulations in numerical relativity

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Quantum results:

Since the discrete theories we generate are constraint free, most of the hard conceptual problems of general relativity are resolved. For instance one can solve the “problem of time” by introducing a relational time in the theory [10]. That is, one promotes all variables to quantum operators and then chooses one of them as a physical “clock” variable and computes relational probabilities for the other variables to take given values when the “clock” variable indicates a certain time. This was attempted in the continuum theory by Page and Wootters [11], but it was shown by Kuchař [12] that the presence of the constraints yields the scheme inviable. The resulting conditional probabilities evolve in terms of the physical clock time in a manner that resembles the usual Schrödinger evolution only if the physical clock variable has no dispersion [13]. In practice this is not possible and therefore the Schrödinger evolution is only approximate. The probabilities evolve in a more complicated way that is not unitary. For small dispersions the corrections are of the Lindblad type [13]. These results are non-controversial, it has been known for a while that imperfect clocks spoil unitarity [14]. Since this effect is fundamental, i.e. it cannot be eliminated in practice, it is good to get a handle on its magnitude. This can be estimated by considering what is the most perfect clock that one can introduced. It turns out that the best clock that can be introduced, following Wigner and others, is a black hole [15]. The requirement that the clock be accurate as an oscillator demands that the mass of the black hole be small. On the other hand, one cannot make it too small or it will evaporate too soon to be useful as a clock. These two limits bracket the possible accuracy of a clock and one gets a compact formula [17] for the ultimate accuracy of a clock $\delta t = t_{\rm Planck}\sqrt[3]{t_{\rm max}/t_{\rm Planck}}$ where $t_m ax$ is the length of time to be measured. With this estimate one can compute how long it will take a quantum system of two energy levels to lose coherence. The off-diagonal elements of the density matrix decay exponentially as a function of time with an exponent $\omega_{12}^2 t^{2/3} t_{\rm Planck}^{4/3}$ where ω_{12} is the Bohr frequency associated with the two energy levels. This is too small an effect to be observed in the laboratory. The only chance of observation may arise with the construction of macroscopic quantum states, like the ones found in Bose-Einstein condensates. Even there, it will require approximately 10^9 atoms for the effect to be visible, and even then one will have to isolate the system from environmental decoherence quite well.

The presence of a fundamental mechanism for loss of coherence of quantum states of gravitational origin has also consequences for quantum computers. The more qubits in the computer, the "more macroscopic" its quantum states are and the larger the loss of coherence. We have estimated [16] that the maximum number of operations (parallel or serial) that a quantum computer of L qubits of size R can carry out is $n \leq \left(\frac{L}{t_{\rm Planck}}\right)^{4/7}\left(\frac{c}{R}\right)^{3/7}$ operations per second.

The fundamental loss of coherence can yield the black hole information paradox invisible [17]. As we argued quantum states lose coherence naturally, albeit at a very small rate. If one collapses a pure state into a black hole and waits for it to evaporate to produce a mixed state, since this process is quite slow, it will actually take longer than the fundamental loss of coherence, at least for macroscopic black holes (for microscopic ones the paradox cannot be formulated either since evaporation assumes one is in the semiclassical limit). Therefore the paradox cannot be really observed in a world with realistic clocks.

A separate development is that the paradigm yields attractive results in the context of quantum cosmologies. Since the lattice spacing is dynamically generated as the universe evolves, if one goes backwards towards the Big Bang, generically, the singularity is avoided [18], since the point fails to fall on the lattice unless one fine tunes the initial data. Quantum mechanically this implies that the singularity has probability zero of being encountered. Moreover, the tunneling to another universe that ensues can be associated with a change in the values of physical constants [19], implementing Smolin's "Life of the cosmos" proposal [20] for the first time in a detailed quantum gravity setting.

Finally, a scheme has been proposed to allow for a better contact with traditional loop quantum gravity [21]. The idea is to discretize time but not space. The resulting theory has no constraints, but one can consistently impose the usual diffeomorphism constraint of loop quantum gravity. The kinematical arena is therefore the same one as in loop quantum gravity, but the dynamics is implemented in an explicit, constraint-free way. The scheme has been successfully tested in BF theories.

Conclusions

We have argued that by concentrating on the properties of the theories that result from discretizing general relativity and demanding that the discrete theories be viable as standalone theories, a new paradigm to study classical and quantum gravity can be created. Among the attractive features is the fact that the paradigm does not involve constraints, a major source of difficulties in general relativity. There is an increasing body of results that imply that the paradigm is viable classically, and an attractive set of predictions at the quantum level. The challenges ahead for the paradigm is to apply it in situations of increasing complexity. The discretization schemes are not optimized for computational efficiency so this will require work. At the quantum level however, this approach provides a readily viable way to implement numerical quantum gravity. The main challenge here will be to show that the continuum limit can be implemented in a satisfactory way. In a sense this will be a way of showing that quantum fields coupled to gravity could be made renormalizable.

It is also remarkable how the paradigm brings together two of the most active areas of research in gravitation (numerical relativity and quantum gravity) that up to now have evolved in separate paths. In the history of science when different fields suddenly coalesce, remarkable results have happened. We are yet to see if this is the case with this new paradigm.

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