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New variables for 1+1 dimensional gravity

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We show that the canonical formulation of a generic action for 1+1-dimensional models of gravity coupled to matter admits a description in terms of Ashtekar-type variables. This includes the CGHS model and spherically symmetric reductions of 3+1 gravity as particular cases. This opens the possibility of discussing models of black hole evaporation using loop representation techniques and verifying which paradigm emerges for the possible elimination of the black hole singularity and the issue of information loss.

I. INTRODUCTION

Gravitational models in 1 + 1 dimensions have proved a fertile ground for testing ideas, in particular ideas about quantization. Examples of these are the treatment of spherically symmetric reduction of 3 + 1 models (see for example [1]) and also models intrinsic to 1 + 1 dimensions like the string-inspired Callan–Giddings–Harvey–Strominger (CGHS) model of black hole evaporation. This model has received renewed attention with the construction of a new paradigm for its interpretation [2, 3]. These recent treatments however, have still been done in terms of traditional quantization techniques. It would be interesting to revisit them using the loop representation. Also recently, we have made progress in treating spherically symmetric reductions of general relativity using loop quantum gravity techniques [4]. In particular we encounter that the singularity inside black holes is eliminated and that the Fock vacuum emerges as a quantum vacuum for a scalar field interacting in spherical symmetry [5]. The success of these techniques in the spherically symmetric reduction of 3 + 1 dimensional gravity strongly suggests that such treatment should be extended to other 1 + 1 dimensional models, like the CGHS model, where the study of Hawking radiation and black hole evaporation is tractable. This allows to discuss the issue of information loss and what paradigm describes better the final fate of a quantum evaporating black hole. In addition to this, 1 + 1 dimensional models are the simplest models where one is faced in full with the problem of dynamics of canonical quantum gravity and the issue of the constraints forming an algebra with structure functions. If one can deal with these problems, one may end up with a full quantum gravitational description of an evaporating black hole, one of the landmark problems of the field. With this goal, in this paper we discuss how to canonically formulate these models using Ashtekar-type variables. There have been treatments of spherically symmetric reductions of 3 + 1 gravity using such types of variables [4, 5, 6, 7, 8, 9], but we will here consider a fairly generic action that encompasses many other 1 + 1-dimensional models of interest.

We start with a quite general action in 1 + 1 dimensions [10]. We will follow the notation of [11],

$$S_{dil} = \int d^2x \sqrt{-|g|} \left(D(\Phi)R(g) + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + U(\Phi) \right). \quad (1)$$

This is the most general diffeomorphism invariant action yielding second order differential equations for the metric g and a scalar dilaton field Φ .

For further analysis, it is convenient to make a conformal transformation $\tilde{g}_{ab} \equiv e^{\rho(\Phi)}g_{ab}$ with

$$\rho = \frac{1}{2} \int^{\Phi} \frac{du}{\frac{dD(u)}{du}} + \text{const.} \quad (2)$$

followed by definition of a new field variable $X^3 \equiv D(\Phi)$.

The superscript notation may appear strange but it is the case that it is the third target space coordinate of a σ -model formulation of the action and as such has been adopted in the literature [11], so we follow it here. With the above transformations the action can be written as,

$$S_{g\text{-gen}} = \int d^2x \sqrt{-|\tilde{g}|} \left\{ X^3 \tilde{R} + V(X^3) \right\} + S_m, \quad (3)$$

where

$$V(z) \equiv \left(\frac{U}{\exp(\rho)} \right) (D^{-1}(z)), \quad (4)$$

and this expression means $U/\exp(\rho)$ evaluated at $\Phi = D^{-1}(z)$. S_m represents the action of additional matter fields one may wish to couple to the model.

There is a subtle issue that needs to be pointed out. In the following calculations we will assume that D has an inverse D^{-1} everywhere on its domain of definition and, for simplicity, we assume that D , D^{-1} , and U are C^∞ . The final result, however, will produce a set of Ashtekar-like variables for action (1) directly, independent of the invertibility or not of D . This is important since both theories are generally non-equivalent. In fact, with an action like (3) it was not originally known how to recover Hawking radiation in the model [12, 13], although more careful analyses have shown how to recover it [14]. Previous treatments of the action (3) with Ashtekar-type variables have been considered by Bojowald and Reyes [15].

Let us now illustrate two particular cases of interest. We start with the spherically symmetric reduction of 3 + 1 gravity in vacuum. We choose as ansatz for the metric $ds^2 = g_{ab}dx^a dx^b + \Phi^2(d\theta^2 + \sin^2(\theta)d\varphi^2)$, where x^0, x^1, θ and φ are coordinates adapted to the spherical symmetry, $a, b = 0, 1$ and $g_{ab}(x^0, x^1)$ is the metric on the x^0, x^1 plane. Inserting this ansatz in the 3 + 1 dimensional Einstein–Hilbert action yields,

$$S_{\text{g-spher}} = \int d^2x \sqrt{-|g|} \left(\frac{1}{4} \Phi^2 R(g) + \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi + \frac{1}{2} \right), \quad (5)$$

where R is the Ricci tensor of the two dimensional metric g^{ab} and $|g|$ is its determinant. In this case the choices are

$$D(\Phi) \equiv \frac{1}{4} \Phi^2, \quad U(\Phi) \equiv \frac{1}{2}. \quad (6)$$

Using this and considering (2), we find for the conformal transformation

$$\rho = \ln(\Phi), \quad (7)$$

and we can identify in the action that $V(X^3) = 1/(4\sqrt{X^3})$. We can then rewrite everything in terms of Φ :

$$g_{ab} = \Phi^{-1} \tilde{g}_{ab} \quad (8)$$

$$\sqrt{-|g|} = \Phi^{-1} \sqrt{-|\tilde{g}|} \quad (9)$$

$$R = \Phi \tilde{R} + \tilde{g}^{ab} \partial_a \partial_b \Phi - \Phi^{-1} \tilde{g}^{ab} \partial_a \Phi \partial_b \Phi. \quad (10)$$

One may choose to minimally couple the model to a scalar field in 3 + 1 dimensions, then one needs to add to the action a term,

$$S_{\text{m-spher}} = - \int d^2x \sqrt{-|g|} \Phi^2 g^{ab} \partial_a f \partial_b f, \quad (11)$$

where f is the scalar field representing matter.

The other particular case of interest is the Callan–Giddings–Harvey–Strominger (CGHS) model. It is given by the above action with the choices

$$D(\Phi) \equiv \frac{1}{8} \Phi^2, \quad U(\Phi) \equiv \frac{1}{2} \Phi^2 \lambda^2 = 4D(\Phi) \lambda^2. \quad (12)$$

where λ^2 is a cosmological constant. The usual form of the CGHS action is obtained by introducing dilaton $\phi = -\ln(\Phi)/(2\sqrt{2})$, yielding

$$S_{\text{g-CGHS}} = \int d^2x \sqrt{-|g|} e^{-2\phi} (R + 4g^{ab} \partial_a \phi \partial_b \phi + 4\lambda^2), \quad (13)$$

We introduce the conformal transformation. Using (12) and considering (2), we find

$$\rho = 2 \ln(\Phi) + c \quad (14)$$

where c is a constant. Choosing $c = -\ln(8)$ one reads off from the action that $V(X^3) = 4\lambda^2$.

The matter part of the action is,

$$S_{\text{m-CGHS}} = - \int d^2x \sqrt{-|g|} g^{ab} \partial_a f \partial_b f \equiv S_{\text{m-1+1}} \quad (15)$$

in which f is the matter scalar field. It should be noted that this form of the matter portion is not restricted to the CGHS model, but corresponds to the coupling to a scalar field in 1 + 1 dimensions no matter what the model. It is only if one decides to couple a scalar field in higher dimensions and then reduce that one gets a different action, as we discussed above. To emphasize this point we will refer to it from now on as S_{m-1+1} .

Let us introduce now a dyadic formulation for the gravitational part of the generic action (3),

$$L_g = -2X_I \epsilon^{ab} (\partial_a e_b^I + \omega_a \epsilon^I_{\ J} e_b^J) - 2X^3 \epsilon^{ab} \partial_a \omega_b + eV(X^3) \quad (16)$$

where X_1 and X_2 are Lagrange multipliers that make the theory torsion-free. ‘‘Internal’’ indices I, J range from 1, 2 and the e_a^I are dyads that we assume are invertible. e is the determinant of the dyad, $e = \frac{1}{2} \epsilon^{ab} \epsilon_{IJ} e_a^I e_b^J$. ϵ_{IJ} and ϵ_{ab} are the two dimensional Levi-Civita symbols and internal indices are raised and lowered with the 1 + 1 dimensional Minkowski metric. The matter part of the Lagrangian of (3) will read for spherically symmetric case:

$$L_{m\text{-spher}} = -4eX^3 \eta^{IJ} e^a_{\ I} e^b_{\ J} \partial_a f \partial_b f \quad (17)$$

and for the generic 1 + 1 case,

$$L_{m-1+1} = -e \eta^{IJ} e^a_{\ I} e^b_{\ J} \partial_a f \partial_b f. \quad (18)$$

The Lagrangian (16) can be rewritten, changing from space to internal indices via,

$$\epsilon^{ab} = -e \epsilon^{IJ} e^a_{\ I} e^b_{\ J}, \quad (19)$$

$$\epsilon_{ab} = -e^{-1} \epsilon_{IJ} e_a^I e_b^J, \quad (20)$$

as,

$$L_g = -2e \epsilon^{KI} e^a_{\ K} \partial_a X_I - 2e e^{aI} X_I \omega_a + 2e X^3 \epsilon^{KL} e^a_{\ K} e^b_{\ L} \partial_a \omega_b + eV(X^3) \quad (21)$$

We now perform a 1 + 1 decomposition of the action. We assume space-time is sliced by surfaces Σ_t parameterized by t . Let g_{ab} be the full space-time metric (strictly speaking in the notation we introduced this should be \tilde{g}_{ab} , we omit the tildes to simplify the notation) and n_a be the unit timelike vector field normal to the hypersurfaces Σ_t ,

$$g_{ab} n^a n^b = -1. \quad (22)$$

Then the space-time metric induces a spatial metric q_{ab} on each Σ_t such that

$$q_{ab} = g_{ab} + n_a n_b, \quad (23)$$

and this metric with one index raised $q_b^a = g^{ac} q_{cb}$ is a projector on the spatial slice. Introducing a vector field $t^a \equiv (\partial/\partial t)^a$, we can define the shift vector $N^a = q^a_b t^b$ and the lapse function $N = -g_{ab} t^a n^b$ such that $n^a = (t^a - N^a)/N$. One also has that $e = N\sqrt{q}$ where q is the determinant of the spatial part of the 1 + 1 metric, i.e. $q = g_{11}$. The spatial components of e_a^I can be written as

$$\begin{aligned} E_a^I &= q_a^b e_b^I \\ &= (g_a^b + n_a n^b) e_b^I \\ &= e_a^I + n_a n^I. \end{aligned} \quad (24)$$

Substituting in the Lagrangian (21) we get,

$$\begin{aligned} L_g = -2N\sqrt{q} \left\{ \right. & \left[E^a_{\ K} - \left(\frac{t^a - N^a}{N} \right) n_K \right] \partial_a^* X^K \\ & + \left[E^{aI} - \left(\frac{t^a - N^a}{N} \right) n^I \right] \omega_a X_I \\ & - X^3 \epsilon^{KL} \left[E^a_{\ K} - \left(\frac{t^a - N^a}{N} \right) n_K \right] \left[E^b_{\ L} - \left(\frac{t^b - N^b}{N} \right) n_L \right] \partial_a \omega_b \\ & \left. - \frac{V(X^3)}{2} \right\} \end{aligned} \quad (25)$$

where the dual of a tensor is defined by $*X^K \equiv \epsilon^{KI} X_I$. The expression can be reduced to

$$\begin{aligned}
L_g = & -2N\tilde{E}^a_K \partial_a *X^K + 2\sqrt{q}n_K t^a \partial_a *X^K - 2\sqrt{q}n_K N^a \partial_a *X^K \\
& -2N\tilde{E}^{aI} X_I \omega_a + 2\sqrt{q}n^I X_I t^a \omega_a - 2\sqrt{q}n^I X_I N^a \omega_a \\
& -2X^3 \epsilon^{KL} \tilde{E}^a_K t^b n_L \partial_a \omega_b - 2X^3 \epsilon^{KL} \tilde{E}^b_L t^a n_K \partial_a \omega_b \\
& + N\sqrt{q}V(X^3).
\end{aligned} \tag{26}$$

where we have used

$$\sqrt{q}E^a_I = \tilde{E}^a_I. \tag{27}$$

Noticing

$$\mathcal{L}_t \omega_b = t^a \partial_a \omega_b + \omega_a \partial_b t^a \tag{28}$$

$$\mathcal{L}_t *X^K = t^a \partial_a *X^K, \tag{29}$$

the gravitational Lagrangian becomes

$$\begin{aligned}
L_g = & -2N\tilde{E}^a_K \partial_a *X^K + 2\sqrt{q}n_K \mathcal{L}_t *X^K - 2\sqrt{q}n_K N^a \partial_a *X^K \\
& -2N\tilde{E}^{aI} X_I \omega_a + 2\sqrt{q}n^I X_I t^a \omega_a - 2\sqrt{q}n^I X_I N^a \omega_a \\
& -2X^3 \epsilon^{KL} \tilde{E}^b_L n_K (\mathcal{L}_t \omega_b - \partial_b(t^a \omega_a)) \\
& + N\sqrt{q}V(X^3).
\end{aligned} \tag{30}$$

which can be rewritten as,

$$\begin{aligned}
L_g = & -2N^* n_I D_1^* X^I - 2\sqrt{q}n_I N^1 D_1^* X^I + 2\sqrt{q}n_I^* \dot{X}^I \\
& + \omega_0 [2\sqrt{q}n_I X^I + \partial_1(2X^3)] + 2X^3 (\dot{\omega}_1) \\
& + N\sqrt{q}V(X^3),
\end{aligned} \tag{31}$$

where the derivative operator D_a is defined by $D_a X_I = \partial_a X_I + \omega_a \epsilon_I^J X_J$. We have chosen adapted coordinates $x^0 = t$, $x^1 = x$ with the standard basis vectors, so $t^0 = 1$ and $t^1 = 0$ and therefore the Lie derivative becomes an ordinary derivative which we denotes by a dot. To derive the above expression we expand \tilde{E}^1_I in terms of some orthonormal vector field in the tangent space of the spatial hypersurface. One can see that the dual of n^I vector field can be a candidate:

$$\begin{aligned}
n_I^ n^I & = \epsilon_{IK} n^K \epsilon^{IL} n_L \\
& = -\delta_K^L n^K n_L \\
& = -n^K n_K \\
& = 1.
\end{aligned} \tag{32}$$

So we are able to expand \tilde{E}^1_I as

$$\tilde{E}^1_I = \tilde{E}^1_{\parallel} *n_I, \tag{33}$$

in which \tilde{E}^1_{\parallel} is the only component of \tilde{E}^1_I with respect to the basis vector field $*n_I$. One can also see that for the \tilde{E}^1_I field:

$$\begin{aligned}
1 & = \eta^{IJ} \tilde{E}^1_I \tilde{E}^1_J \\
& = \eta^{IJ} \tilde{E}^1_{\parallel} *n_I \tilde{E}^1_{\parallel} *n_J \\
& = (\tilde{E}^1_{\parallel})^2,
\end{aligned} \tag{34}$$

and thus

$$\tilde{E}^1_{\parallel} = 1. \tag{35}$$

Using (33) and (35), we can rewrite \tilde{E}^1_I as

$$\begin{aligned}
\tilde{E}^1_I & = \tilde{E}^1_{\parallel} *n_I \\
& = *n_I.
\end{aligned} \tag{36}$$

Based on this we can arrive at a useful observation:

$$\begin{aligned}
\epsilon^{KL}\tilde{E}^1{}_{Ln_K} &= \epsilon^{KL}\tilde{E}^1{}_{\parallel}{}^*n_Ln_K \\
&= \epsilon^{KL}{}^*n_Ln_K \\
&= \epsilon^{KL}\epsilon_{LJ}n^Jn_K \\
&= -\epsilon^{LK}\epsilon_{LJ}n^Jn_K \\
&= -(-\delta_J^K)n^Jn_K \\
&= n^Kn_K \\
&= -1.
\end{aligned} \tag{37}$$

and using this we get

$$\begin{aligned}
2X^3\epsilon^{KL}\tilde{E}^1{}_{Ln_K}\partial_1\omega_0 &= \partial_1(2X^3\epsilon^{KL}\tilde{E}^1{}_{Ln_K}\omega_0) - \omega_0\partial_1(2X^3\epsilon^{KL}\tilde{E}^1{}_{Ln_K}) \\
&= -\partial_1(2X^3\omega_0) + \omega_0\partial_1(2X^3).
\end{aligned} \tag{38}$$

Going back to the Lagrangian (31) one can immediately identify the canonical variables ${}^*X^I$, $I = 1, 2$ and ω_1 , and their canonical momenta,

$$P_I = \frac{\partial L_g}{\partial {}^*\dot{X}^I} = 2\sqrt{q}n_I, \tag{39}$$

$$P_3 = \frac{\partial L_g}{\partial \dot{\omega}_1} = 2X^3. \tag{40}$$

One can then rewrite the Lagrangian in terms of the canonical variables,

$$\begin{aligned}
L_g &= -2N\epsilon_{IJ}\frac{P^J}{\|P\|}D_1{}^*X^I - P_I N^1 D_1{}^*X^I + P_I{}^*\dot{X}^I \\
&\quad + \omega_0 [P_I\epsilon^{IJ}{}^*X_J + \partial_1(P_3)] + P_3(\dot{\omega}_1) \\
&\quad + N\frac{\|P\|}{2}V(X^3).
\end{aligned} \tag{41}$$

where

$$\|P\|^2 = -\eta^{IJ}P_IP_J = -4q\eta^{IJ}n_In_J = 4q. \tag{42}$$

For the gravitational part one can therefore write the total Hamiltonian in the generic case,

$$\begin{aligned}
H_{\text{gen}} &= N\left(2\epsilon_{IJ}\frac{P^J}{\|P\|}D_1{}^*X^I - \frac{\|P\|}{2}V(P^3)\right) \\
&\quad + N^1(P_ID_1{}^*X^I) - \omega_0(P_I\epsilon^{IJ}{}^*X_J + \partial_1(P_3)).
\end{aligned} \tag{43}$$

Let us now turn to the matter Lagrangians. Denoting $\partial_1 f \equiv f'$ and $\partial_t f \equiv \dot{f}$ the matter part of the Lagrangian for the spherical symmetric reduction of 3 + 1 gravity can be written as,

$$L_{m\text{-spher}} = 4\sqrt{q}X^3\left\{-\frac{N}{q}f'^2 + \frac{1}{N}\dot{f}^2 - \frac{2}{N}N^1\dot{f}f' + \frac{1}{N}(N^1)^2f'^2\right\}. \tag{44}$$

Now it is easy to read the canonical variable f and its conjugate being

$$P_f = \frac{\partial L_{m\text{-spher}}}{\partial \dot{f}} = \frac{8\sqrt{q}X^3}{N}[\dot{f} - N^1f']. \tag{45}$$

which leads to

$$H_{m\text{-spher}} = \frac{4NP_3f'^2}{\|P\|} + \frac{N(P_f)^2}{4\|P\|P_3} + N^1f'P_f. \tag{46}$$

For the generic 1 + 1 case, the matter Lagrangian differs from the one we just considered in a factor $4X^3$, so very straightforwardly one gets,

$$H_{m-1+1} = \frac{2Nf'^2}{\|P\|} + \frac{N(P_f)^2}{2\|P\|} + N^1 f' P_f \quad (47)$$

The total Hamiltonian including matter for the generic 1 + 1 case is,

$$\begin{aligned} H_{\text{gen}} = & N \left(2\epsilon_{IJ} \frac{P^J}{\|P\|} D_1^* X^I - \frac{\|P\|}{2} V(P^3) + \frac{2f'^2}{\|P\|} + \frac{(P_f)^2}{2\|P\|} \right) \\ & + N^1 (P_I D_1^* X^I + f' P_f) - \omega_0 (P_I \epsilon^{IJ} X_J + \partial_1(P_3)). \end{aligned} \quad (48)$$

The generic analysis can be carried out only up to this point since the conformal transformation leading from the original metric variables to those with tildes given by equation (2) involves an arbitrary function $D(\Phi)$. To complete the construction of the Ashtekar-type variables one needs to specify such function.

Let us now introduce Ashtekar-like variables for both cases, starting with the spherical reduction of 3 + 1 gravity. We first notice that in this case $E^x = \Phi^2 = 4X^3 = 4D(\Phi)$ with E^x the densitized triad in the radial direction, as can be readily seen from the form of the spherically symmetric metric. On the other hand $q = \tilde{g}_{11} = (E^\varphi)^2 / \sqrt{E^x}$. The components 1, 2 of the normal vector n_I form a vector in the ‘‘transverse’’ space to the radial direction that is normalized so they can be parameterized by an angle η ,

$$n_1 = \cosh(\eta) \quad (49)$$

$$n_2 = \sinh(\eta). \quad (50)$$

We can now introduce the densitized triad in the φ direction by using its relation to the determinant of the three metric,

$$\sqrt{q} = \frac{E^\varphi}{(E^x)^{\frac{1}{4}}} = \frac{\|P\|}{2}. \quad (51)$$

We also note from (39) and (51) that

$$P_1 = n_1 \|P\| = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \cosh(\eta) \quad (52)$$

$$P_2 = n_2 \|P\| = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \sinh(\eta) \quad (53)$$

$$P_3 = \frac{E^x}{2}. \quad (54)$$

and with the above relations we can motivate a type II canonical transformation that will leave us with canonical variables E^x, E^φ, η and their canonically conjugates which we will call A_x, K_φ, Q_η . The generating function is

$$F(q, P) = {}^*X^1 \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \cosh(\eta) + {}^*X^2 \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \sinh(\eta) + \omega_1 \frac{E^x}{2}, \quad (55)$$

and it leads to the following expressions for the new canonical variables,

$$Q_\eta = \frac{\partial F}{\partial \eta} = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} ({}^*X^1 \sinh(\eta) + {}^*X^2 \cosh(\eta)) = -{}^*X_1 P_2 + {}^*X_2 P_1 \quad (56)$$

$$K_\varphi = \frac{\partial F}{\partial E^\varphi} = \frac{2({}^*X^1 \cosh(\eta) + 2{}^*X^2 \sinh(\eta))}{(E^x)^{\frac{1}{4}}} = \frac{-{}^*X_1 P_1 + {}^*X_2 P_2}{E^\varphi} \quad (57)$$

$$A_x = \frac{\partial F}{\partial E^x} = -\frac{E^\varphi}{2(E^x)^{\frac{3}{4}}} ({}^*X^1 \cosh(\eta) + {}^*X^2 \sinh(\eta)) + \frac{\omega_1}{2} \quad (58)$$

$$= \frac{{}^*X_1 P_1 - {}^*X_2 P_2}{4E^x} + \frac{\omega_1}{2} \quad (59)$$

$$= -\frac{E^\varphi K_\varphi}{4E^x} + \frac{\omega_1}{2} \quad (60)$$

where we have used $*X^1 = -*X_1$ and $*X^2 = *X_2$. We still need to find expressions for $*X_1$ and $*X_2$ in terms of these new variables. One can see that,

$$*X_1 = \frac{Q_\eta(E^x)^{\frac{1}{4}} \sinh(\eta)}{2E^\varphi} - \frac{K_\varphi(E^x)^{\frac{1}{4}} \cosh(\eta)}{2}, \quad (61)$$

$$*X_2 = \frac{Q_\eta(E^x)^{\frac{1}{4}} \cosh(\eta)}{2E^\varphi} - \frac{K_\varphi(E^x)^{\frac{1}{4}} \sinh(\eta)}{2}. \quad (62)$$

We are now ready to express the total Hamiltonian in terms of the new variables,

$$\begin{aligned} H_{\text{spher}} = & N \left[\frac{1}{2} (E^x(x))^{\frac{1}{4}} E^\varphi(x) (f'(x))^2 + \frac{1}{4} \frac{Q_\eta(x) E^{x'}(x)}{(E^x(x))^{\frac{3}{4}} E^\varphi(x)} - \frac{(E^x(x))^{\frac{1}{4}} Q_\eta(x) E^{\varphi'}(x)}{(E^\varphi(x))^2} \right. \\ & + \frac{(E^x(x))^{\frac{1}{4}} Q'_\eta(x)}{E^\varphi(x)} - (E^x(x))^{\frac{1}{4}} K_\varphi(x) \eta'(x) - 2(E^x(x))^{\frac{1}{4}} A_x(x) K_\varphi(x) \\ & \left. - \frac{1}{2} \frac{E^\varphi(x) (K_\varphi(x))^2}{(E^x(x))^{\frac{3}{4}}} - \frac{1}{2} \frac{E^\varphi(x)}{(E^x(x))^{\frac{3}{4}}} + \frac{1}{4} \frac{(P_f(x))^2}{E^\varphi(x) (E^x(x))^{\frac{1}{4}}} + \frac{(E^x(x))^{\frac{5}{4}} (f'(x))^2}{E^\varphi(x)} \right] \\ & + \omega_0 \left[Q_\eta(x) - \frac{1}{2} E^{x'}(x) \right] \\ & + N^1 \left[-Q_\eta(x) \eta'(x) + \frac{1}{4} \frac{K_\varphi(x) E^{x'}(x) E^\varphi(x)}{E^x(x)} + K'_\varphi(x) E^\varphi(x) - 2A_x(x) Q_\eta(x) \right. \\ & \left. - \frac{1}{2} \frac{E^\varphi(x) K_\varphi(x) Q_\eta(x)}{E^x(x)} + P_f(x) f'(x) \right] \quad (63) \end{aligned}$$

and we readily distinguish a Hamiltonian and diffeomorphism constraints and a Gauss law. One can proceed further by solving Gauss' law $Q_\eta(x) = \frac{1}{2} E^{x'}(x)$ and defining a new variable $K_x(x) = \frac{1}{2} \eta'(x) + A_x(x)$, where one is left with a model with a Hamiltonian and diffeomorphism constraint and with canonical pairs E^x, K_x and E^φ, K_φ . We will not repeat the calculation here since it is already present in the literature [4, 9].

Let us now turn our attention to the CGHS model. The construction is virtually similar, except for small details. We first introduce E^x and the angle η ,

$$X^3 = D(\Phi) = \frac{E^x}{8} \quad (64)$$

$$\sqrt{q} = E^\varphi = \frac{\|P\|}{2} \quad (65)$$

$$n_1 = \cosh(\eta) \quad (66)$$

$$n_2 = \sinh(\eta) \quad (67)$$

and

$$\|P\| = 2E^\varphi \quad (68)$$

and from (39) and (65)

$$P_1 = n_1 \|P\| = 2E^\varphi \cosh(\eta), \quad (69)$$

$$P_2 = n_2 \|P\| = 2E^\varphi \sinh(\eta), \quad (70)$$

$$P_3 = \frac{E^x}{4} \quad (71)$$

and the generating function for the type II canonical transformation is,

$$F(q, P) = *X^1 2E^\varphi \cosh(\eta) + *X^2 2E^\varphi \sinh(\eta) + \omega_1 \frac{E^x}{4}, \quad (72)$$

The new canonical variables are then

$$Q_\eta = \frac{\partial F}{\partial \eta} = 2E^\varphi (*X^1 \sinh(\eta) + *X^2 \cosh(\eta)) = -*X_1 P_2 + *X_2 P_1 \quad (73)$$

$$K_\varphi = \frac{\partial F}{\partial E^\varphi} = 2*X^1 \cosh(\eta) + 2*X^2 \sinh(\eta) = \frac{-*X_1 P_1 + *X_2 P_2}{E^\varphi} \quad (74)$$

$$A_x = \frac{\partial F}{\partial E^x} = \frac{\omega_1}{4} \quad (75)$$

and the total Hamiltonian is,

$$\begin{aligned}
H_{\text{CGHS}} = & N \left[\frac{(f'(x))^2}{E^\varphi(x)} - \frac{Q_\eta(x)E^{\varphi'}(x)}{(E^\varphi(x))^2} + \frac{Q'_\eta(x)}{E^\varphi(x)} - K_\varphi(x)\eta'(x) \right. \\
& \left. - 4A_x(x)K_\varphi(x) - 4E^\varphi(x)\lambda^2 + \frac{1}{4} \frac{(P_f(x))^2}{E^\varphi(x)} \right] \\
& + \omega_0 \left[Q_\eta(x) - \frac{1}{4} E^{x'}(x) \right] \\
& + N^1 \left[-Q_\eta(x)\eta'(x) + K'_\varphi(x)E^\varphi(x) - 4A_x(x)Q_\eta(x) + P_f(x)f'(x) \right]
\end{aligned} \tag{76}$$

where again we easily distinguish the Gauss law, Hamiltonian and diffeomorphism constraint. We can gauge fix Gauss' law $Q_\eta(x) = \frac{1}{4}E^{x'}(x)$ and defining $K_x(x) = \frac{1}{4}\eta'(x) + A_x(x)$ we are left with a total Hamiltonian,

$$\begin{aligned}
H_{\text{CGHS}} = & N\mathcal{H} + N^1\mathcal{D} \\
= & N \left[-\frac{1}{4} \frac{E^{x'}(x)E^{\varphi'}(x)}{(E^\varphi(x))^2} + \frac{1}{4} \frac{E^{x''}(x)}{E^\varphi(x)} - 4K_\varphi(x)K_x(x) - 4E^\varphi(x)\lambda^2 \right. \\
& \left. + \frac{(f'(x))^2}{E^\varphi(x)} + \frac{1}{4} \frac{(P_f(x))^2}{E^\varphi(x)} \right] \\
& + N^1 \left[-E^{x'}(x)K_x(x) + E^\varphi(x)K'_\varphi(x) + P_f(x)f'(x) \right]
\end{aligned} \tag{77}$$

We therefore have developed a technique for constructing Hamiltonian formulation for generic models of gravity in 1 + 1 dimensions in terms of Ashtekar variables. This includes the CGHS black hole model. These will be the point of departure for future investigations that will probe the emergence of a paradigm in which black hole singularities are eliminated and the information loss problem in evaporating black holes can be treated using the loop representation. The techniques are ready. The following steps would be to polymerize the above expressions and a study of the resulting semiclassical theory to study modifications or perhaps the elimination of the singularity. This can be followed by studies in the full quantum theory using the uniform discretization procedure, as has been carried out in the spherical case already [5], where the vacuum has been identified and progress is being made on computing propagators.

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