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Emergence of string-like physics from Lorentz invariance in loop quantum gravity*

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Abstract

We consider a quantum field theory on a spherically symmetric quantum space time described by loop quantum gravity. The spin network description of space time in such a theory leads to equations for the quantum field that are discrete. We show that to avoid significant violations of Lorentz invariance one needs to consider specific non-local interactions in the quantum field theory similar to those that appear in string theory. This is the first sign that loop quantum gravity places restrictions on the type of matter considered, and points to a connection with string theory physics.

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There has been recent progress in studying quantum field theory on a quantum space-time in loop quantum gravity [1]. In particular spherically symmetric quantum space-times were exactly solved and can be used as a background to study quantum matter fields living on black hole space times[2]. If one considers background quantum states that are peaked around a definite value of the ADM mass, the main effect of the quantum nature of the background is that the equations that result for the field theory are discrete, as if it had been placed on a lattice [3]. Except that in this case the lattice is not a computational tool, but is fundamental in nature, it is given by the discreteness of space-time in loop quantum gravity. In particular, in spherical symmetry one has that the areas of the spheres of symmetry are given by an integer multiple of the Planck area. The presence of a lattice structure leads to violations of Lorentz invariance. How to recover the approximate continuum behavior of the quantum field theory in regions of small curvature, for low energies and for interacting quantum field theories has to be addressed. It is well known that although the violations appear at Planck scale only, Collins *et al.* [4] have argued that as soon as one considers interacting quantum field theories and certain diagrams requiring renormalization, the violations become large and experimentally unacceptable. In this essay we would like to argue that to avoid such problems, one needs to consider interacting quantum field theories with non-local interactions, of a kind similar to those that appear in string field theory [6].

As we shall see, some degree of non-locality is expected since the background quantum space-time will generically be in a state that is the superposition of different values of the ADM mass. This is tantamount to consider superpositions of lattices, which leads to spatial non-locality. To preserve Lorentz invariance, however, one is led to consider non-locality in space-time as those considered in string theory.

Let us briefly recall our previous quantum treatment of vacuum spherically symmetric space-times [2]. The quantum states that solve the diffeomorphism and Hamiltonian constraints are based on one dimensional spin networks. A basis is labeled by a graph g , a vector of valences \vec{k} and the value of the ADM mass, $|\tilde{g}, \vec{k}, M\rangle$, where \tilde{g} is the equivalence class of graphs under diffeomorphisms of g . The integer values of the vector of valences characterize the value of the area of the spherical surfaces of symmetry in terms of Planck units. On these states one can realize the Hamiltonian of a scalar matter (test) field as a Dirac observable acting on the vacuum gravitational states [3]. The expectation value of the Hamiltonian taken on the vacuum gravitational states can be used to construct the

equations for the matter field. In the particular case in which the vacuum quantum state is peaked in the ADM mass, the resulting equations for matter look like discrete versions of the continuum equations for the field. The discreteness is induced by the background quantum geometry.

Generically, the background quantum state will not be in an eigenstate of the ADM mass. A spread of the ADM mass translates itself in a fuzziness of the resulting discrete quantum field theory. To see this, we notice that the matter field lives at the vertices of the spin network of the background quantum state. It is therefore characterized by a vector of values $\vec{\phi}$ representing the values of the field at the vertices of the spin network. The complete quantum state for gravity and matter is given by $|\tilde{g}, \vec{k}, M, \vec{\phi}\rangle$. To make contact with the traditional picture of quantum field theory on a classical space-time, one needs to make several assumptions [2, 3], and in particular to identify the components of the vector $\vec{\phi}$ with values of the fields at particular coordinate points $\phi(r)$. Such identification will generically depend on the complete quantum state of matter and gravity. If such a state is in a superposition of values of the mass (and also generically, of the valences \vec{k}) this will imply that each value $\phi(r)$ will correspond to a superposition of components of $\vec{\phi}$. For instance, if we consider a Gaussian spread in the mass, this will imply a Gaussian superposition in the fields of different points in the spin network, which in Fourier space translates into the fields picking up a factor $\exp(-\sigma(\Delta M) (\vec{p})^2)$ with $\sigma(\Delta M)$ a function dependent on the width of the superposition in the mass, which we take to be of Planck scale $\Delta M \sim \ell_{\text{Planck}}$. To derive the factor in three dimensions one needs to expand the plane waves one uses to compute the Feynman propagators in terms of spherical waves. One can show that the fuzziness in the radial direction induces one in the angular directions.

Such factors can help with the violations of Lorentz invariance due to the presence of the discreteness, although they do not solve them completely. One will need an additional ingredient. To see this, let us start by briefly recalling the argument of Collins *et al.* [4]. They consider a model consisting of a Yukawa coupling of a scalar field and a fermion, but the essence of the argument applies to generic interacting quantum field theories. They study diagrams that involve integrals over internal momentum lines. Such integrals diverge and have to be renormalized. They focus on a particular quantity, the self-energy and compute its second derivative with respect to p_0 and with respect one of the spatial components of the momentum, for instance p_1 and subtract them, evaluating them in $p^\mu = 0$. If the quantity

is Lorentz invariant (i.e. function of $p^\mu p_\mu$), such subtraction should vanish. Computing the self energy with the usual propagators in the continuum the result is indeed zero with a suitable regularization procedure that respects Lorentz invariance. However, they show that if one considers propagators that violate Lorentz invariance, even if the violation only happens at very high energies, the result is non-vanishing and in fact it is bounded below by finite quantity that would lead to incompatibilities with experimental evidence.

If one were to repeat Collins *et al.*'s calculation for the quantum field theory on a quantum space time we are considering, the fact that effectively the resulting theory is discrete implies that dispersion relations for the fields are those of a lattice theory and therefore not Lorentz invariant. A straightforward calculation [5] shows that the second derivatives of the self energy do not cancel and one has large violations of Lorentz invariance. What about the extra exponential factors in the momentum of the field we noticed? Those factors apparently would help since they tend to cutoff large values of the momentum and therefore confine the calculation to a regime where the dispersion relation of the lattice theory is approximately Lorentz invariant. However, by imposing a cutoff only in the spatial part of the momentum, one is further violating Lorentz invariance (the integral in p^0 is unbounded while the other is damped by the Gaussian factor) and one can show that the second derivatives do not cancel, again implying unacceptably large violations of Lorentz invariance. This was actually explicitly studied by Collins *et al.* [4].

As we stated, we need an additional ingredient to restore Lorentz invariance at low energies. This additional ingredient is to consider a theory with a non-local interaction that will create a similar bound as the one we had in spatial momenta in the time component of the momentum. For simplicity, let us consider a $\lambda\phi^4/4!$ theory. We replace the interaction in momentum space by,

$$\frac{\lambda}{4!} \left[\exp \left(-\alpha^2 (p_0^2 - (\vec{p})^2)^2 \right) \phi(p_0, \vec{p}) \right]^4, \quad (1)$$

where α is a function of ΔM . This interaction would correspond in position space to a non-local interaction. There are two requirements guiding us: the interaction should be Lorentz invariant and the exponential nature of the factor is to make it compatible with the non-locality discussed before. This does not determine the interaction uniquely, but significantly constrains its functional form. The extra factors present imply that the integrals in momentum space are confined to regions close to the light cone. Combined

with the Gaussian factors in spatial momentum we discussed before, they imply that both the integrals on the spatial and temporal components of the momentum are confined to regions where the dispersion relations of the discrete treatment are approximately Lorentz invariant. One can show that the calculation of the second derivatives of the self energy considered by Collins *et al.* yields a result that can be made arbitrarily small by choosing appropriate functions of the mass width α and σ (see appendix). Given that the lattice separations imposed by the quantization of area in loop quantum gravity are very small, Lorentz invariance holds very accurately up to energies considerably higher than grand unification energies. The resulting effective theory one recovers is the usual $\lambda\phi^4/4!$ theory.

The non-local exponential types of interactions we are considering have been extensively studied in the context of string theories [6]. In general non-local theories have problems related to ghosts and the Ostrogradsky instability. However, there exist more benign non-local theories that arise as effective field theories in string theory and the type of theory we are considering here is of that class. It should be noted that this is the first instance in which loop quantum gravity imposes restrictions on the matter content of the theory. Up to now loop quantum gravity, in contrast to supergravity or string theory, did not appear to impose any restrictions on matter. Here we are seeing that in order to be consistent with Lorentz invariance at small energies, limitations on the types of interactions that can be considered arise. The limitations include having to involve constants that are determined by properties of the geometry. One can also expect that the values of the coupling constants will relate to their bare values through expressions involving properties of the geometry as well, like fluctuations in the Schwarzschild radius.

Summarizing, we have argued that in order to have Lorentz invariance at low energies in quantum field theories on the quantum space-times that arise in loop quantum gravity one needs to start with matter fields that have interactions of the type that arise in effective theories stemming from string theory. This implies a significant restriction on kind of interactions in the matter content of the theory and opens the possibility for contacts between the physics of loop quantum gravity and string theory.

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I. APPENDIX: A MODEL CALCULATION

The calculation of the second derivatives of the self energy for a discrete theory with the weighting functions we consider is lengthy. To illustrate its behavior we exhibit here a simple example that captures what is going on. Consider the following integral in two dimensions,

$$I = \int_{-\infty}^{\infty} dx dy \frac{x^2 - y^2}{\left(1 + x^2 + \frac{y^2}{1+\sigma y^2}\right)^2}. \quad (2)$$

If $\sigma = 0$ the denominator is invariant under rotations and the integral vanishes. This would be the analogue of the difference of the derivatives of the self-energy diagram calculation in the continuum when Lorentz invariance holds. The case σ different from zero would correspond to the calculation on the lattice, where Lorentz invariance is broken, in this example represented by the breakage of rotational invariance. σ would play the role of lattice spacing. When it is not zero, the different behavior in x and y for large values implies the integral does not vanish by a significant amount. However, if one considers the integral with weighting factors similar to the ones considered in this essay,

$$I = \int_{-\infty}^{\infty} dx dy \frac{x^2 - y^2}{\left(1 + x^2 + \frac{y^2}{1+\sigma y^2}\right)^2} e^{-\alpha x^2} e^{-\beta(x^2+y^2)} \quad (3)$$

with the first weighing factor representing the one stemming from fuzziness and the second representing the non-local interaction (in this case rotationally invariant, in the case of interest, Lorentz invariant), one can see that with suitable choices of α and β one can make the integral as small as desired. This is due to the exponentials cutting off the integral in the region (large values of x, y) that yielded the non-vanishing contributions. More precisely one needs $\alpha \sim \beta$ and both larger than σ .

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