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INTERSUBBAND-CYCLOTRON COMBINED RESONANCE IN A SURFACE SPACE-CHARGE LAYER

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ABSTRACT

In a magnetic field tilted with respect to the surface of Si, Beinvogl and Koch[1] observed combined resonance transitions, resulting from a coupling of Landau levels and subband states. Because of some unexplained features in the observations, we analyze what one should expect on theoretical grounds. We conclude that unexplained discrepancies between theory and experiment still exist.

DISCLAIMER

A strong electric field, ε, applied normal to a semiconductor surface gives rise to quantized electron motion in this direction, with the result that two-dimensional electric subbands are formed, with energies E_n. If $\varepsilon = \varepsilon_z$, and if a magnetic field H is also applied in the same direction then each subband is further quantized into discrete Landau levels so that the energy becomes $E_{n,N} = E_n + (N+1/2)\hbar\omega_c$, where $\omega_c = (eH_z/cm_{//})$ is the cyclotron frequency, and where $m_{//} = 0.1905\ m_o$ is the effective mass in the direction parallel to the surface, m_o denoting the free electron mass.

In a recent experiment, Beinvogl and Koch [1] investigated electrons on Si(1,0,0), in the presence of a magnetic field tilted with respect to the sample surface (H_y and H_z components), and observed combined resonance transitions because of a coupling of Landau levels and subband states. Surprisingly, they found that the sum of the separations for the $\Delta N = 1$ and $\Delta N = -1$ transitions is $\{0.7 - (-1.6)\}\hbar\omega_c = 2.3\hbar\omega_c$ i.e. 15% higher than the expected result of $2\hbar\omega_c$.

Prior theoretical work on this problem by Ando [2] reached the conclusion that

$$E_{n,N} = E_n + \Delta E_n + E_N \quad , \tag{1}$$

where

$$E_N = (N+\tfrac{1}{2})\hbar\omega_c \quad , \tag{2}$$

$$\Delta E_n = \frac{e^2}{2m_{//}c^2} H_y^2 \left[(z^2)_{nn} - (z_{nn})^2 \right] . \tag{3}$$

Thus we get the characteristic energy changes of $(\hbar\omega_c)\Delta N$ corresponding to ΔN transitions. As shown by Ando [2] this conclusion is not affected by

the inclusion of many-body and other effects. The latter affect the difference in the positions of the main ($\Delta N = 0$) and a combined ($\Delta N \neq 0$) resonance peak but does not affect the difference in the positions of two combined resonance peaks. A basic assumption made by Ando was to treat H_y as a perturbation so that its influence on the z-part of the wave function is neglected. Recently Ando [3] carried out a more detailed investigation without this restriction but it is clear (see figure 9 of reference 3) that discrepancies between experiment and theory still exist.

It is our purpose here to return to the original perturbation analysis to investigate more precisely the extent of its validity. As we shall see/ our conclusion is that it should be very good as far as an analysis of the Beinvogl-Koch observations are concerned. To this end we will calculate ΔE_n explicitly by using the simplest realistic model for the z-potential, $V(z)$. Following Stern [4] we use the triangular-potential approximation i.e. $V(z) = e\mathcal{E}z$ for $z>o$, with an infinite barrier for $z<o$. The corresponding wave-function is an Airy function, from which it readily follows that[4]

$$E_n \simeq (\hbar^2/2m_\perp)^{1/3} \left[\frac{3}{2}\pi e \mathcal{E} (n + \frac{3}{4})\right]^{2/3} , \tag{4}$$

and

$$z_{nn} = 2E_n/3e\mathcal{E} \; ; \; \langle z^2 \rangle_{nn} = \frac{6}{5} (z_{nn})^2 , \tag{5}$$

and where $m_\perp = 0.916\, m_o$ is the effective mass in the z direction.

Hence, from equations (3) and (5), we obtain

$$\Delta E_n = \frac{2}{45} \frac{e^2 H_y^2}{m_\parallel c^2} \left(\frac{E_n}{e\mathcal{E}}\right)^2 , \tag{6}$$

and thus

$$E_n + \Delta E_n = E_n \left\{ 1 + \frac{2}{45} \left(\frac{H_y}{\varepsilon} \right)^2 \frac{E_n}{m_{//} c^2} \right\}. \qquad (7)$$

The magnitude of the H_y^2 term inside the braces, compared to unity, will be a measure of goodness of the perturbation approach. If it is << 1 then the perturbation analysis should be very good. Since $E_n \sim \varepsilon^{2/3}$ we see that this H_y^2 term $\sim \varepsilon^{-4/3}$ and thus it increases with increasing n and decreasing ε.

Now the observations were carried out in a sweep of the gate voltage for fixed infra-red energies $\hbar\omega$ = 10.45 and 15.81 meV. The values selected for H_z were 5T and 3.5T, so that the corresponding values of $\hbar\omega_c$ are 3.0 meV and 2.1 meV, respectively. The values of H_y ranged from 0 to 6T. Typical values for E_n and ε (chosen to maximize the H_y^2 term inside the braces) are 10 meV and 10^5 V/cm (3.3×10^2 stat-volt/cm), respectively. Also, $m_{//} c^2 = 9.7 \times 10^4$ eV. Thus, choosing the maximum value of H_y used in the observations, $(H_y/\varepsilon)^2 \sim (6 \times 10^4 / 3.3 \times 10^2)^2 \sim 3.3 \times 10^4$ and $(E_n / m_{//} c^2) \sim 10^{-7}$. As a result, we conclude that ΔE_n is typically $\sim 10^{-4}$ times smaller than E_n and thus negligible. It is also less than 10^{-3} times the $\hbar\omega_c$ term. In other words, we are led to the basic conclusion that ΔN transitions should give the familiar $(\hbar\omega_c)$ ΔN energy changes.

Since the values we have used for H_y and H_z are comparable the question remains as to why H_z makes the dominant contribution to the energy. The reason is that H_z makes the dominant contribution to the energy in the x-y plane. On the other hand, H_y affects the motion in the z and x directions. In the z direction its effect is overwhelmed by the electric field effects. With regard to its effect on the x motion we note that the basic Hamiltonian contains a $P_x H_y$ term, (in addition to a H_y^2 term) where P_x is the momentum

in the x-direction. However, this term does not contribute a linear H_y contrubution to the energy (unlike the H_z term, whose contribution to the energy is linear in H_z) for the simple reason that P_x averages to zero in lowest order (i.e. absence of H_y). Thus the contribution of $P_x H_y$ to the energy is of order H_y^2 and thus higher order than might have been first surmised.

As already pointed out by many authors, the choice of a more realistic V(z), to include polarization, excitonic, and many-body effects, has important consequences. In particular the latter effects give rise to a shift in the energy of the pure subband resonance ($\Delta N = 0$). However, as mentioned above, such effects were shown to be irrelevant [2] to the discussion of the difference in the positions of two combined resonance peaks, which is the main theme of this communication. Our choice of V(z) is also the dominant contribution to the real potential.

We conclude that perturbation theory should give very good results. This conclusion is also implicit in the work of others but no explicit calculations have been presented - as we have done above - showing the extent of the validity of the results obtained from perturbation theory.

We conclude that a definite discrepancy still exists between theoretical expectations and the observations of Beinvogl and Koch. This could be due to a missing ingredient in the theoretical analysis or else a mis-interpretation of the observations. In this respect, we note that Beinvogl and Koch themselves speculate on "... a likely source of the discrepancy..." as being their estimate of the actual energies involved.

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