Towards automation of forensic facial reconstruction

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TOWARDS AUTOMATION OF FORENSIC FACIAL RECONSTRUCTION

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering in The Department of Mechanical Engineering

by

Hemant Narendra Khatod
B.E., Marathwada Institute of Technology, Dr. B.A. Marathwada University, 2001
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Abstract

Forensic facial reconstruction is a blend of art and science thus computerizing the process leads to numerous solutions. However, complete automation remains a challenge.

This research concentrates on automating the first phase of forensic facial reconstruction which is automatic landmark detection by model fitting and extraction of feature points. Detection of landmarks is a challenging task since the skull orientation in a 3D scanned data cloud is generally arbitrary and unknown. To address the issue, well defined skull and mandible models with known geometric structure, features and orientation are (1) aligned and (2) fit to the scanned data. After model fitting is complete, landmarks can be extracted, within reasonable tolerance, from the dataset.

Several methods exist for automatic registration (alignment); however, most suffer ambiguity or require interaction to manage symmetric 3D objects. A new alternative 3D model to data registration technique is introduced which works successfully for both symmetric and non-symmetric objects. It takes advantage of the fact that the model and data have similar shape and known geometric features. Therefore, a similar canonical frame of reference can be developed for both model and data. Once the canonical frame of reference is defined, the model can be easily aligned to data by a euclidian transformation of its coordinate system.
Once aligned, the model is scaled and deformed globally to accommodate the overall size of the object and bring the model in closer proximity to the data. Lastly, the model is deformed locally to better fit the scanned data. With fitting completed, landmark locations on the model can be utilized to isolate and select corresponding landmarks in the dataset.

The registration, fitting and landmark detection techniques were applied to a set of six mandible and three skull body 3D scanned datasets. Results indicate the canonical axes formulation is a good candidate for automatic registration of complex 3D objects. The alternate approach posed for deformation and surface fitting of datasets also shows promise for landmark detection when using well constructed NURBS models. Recommendations are provided for addressing the algorithms limitations and to improve its overall performance.
Chapter 1. Introduction

The objective of forensic facial reconstruction is to reproduce, from a skull, a face with sufficient likeness of an individual that it can facilitate victim identification when there are no other means available. This technique has been used since 1895 [Grüner, 1993]. The process has been refined over the years and a lot of statistical data is available which helps in predicting the age, gender, and race from the skull. However, the process is very time consuming. Modification of the reconstructed face is a tedious task and often the original skull must be recovered for evidence, thereby destroying the reconstructed face.

Recent advances in computer graphics, and the limitations of manual techniques motivated researchers to work towards automation of forensic facial reconstruction. Successful attempts have been made to computerize portions of forensic facial reconstruction process; however, complete automation of the process remains a challenge.

The basic sequence of step in manual facial reconstruction are:

1. A skull is cleaned and prepared for facial reconstruction;
2. Skull measurements are recorded and based on this information, a forensic analyst decides the age, the gender and the race of the victim;
3. Using the age, gender and race information, tissue depths at landmarks are extracted from a database;
4. Dowels with the proper tissue depth are placed on landmarks;
5. A forensic analyst reconstructs a face on the skull using clay to fill between the landmarks.
Each task mentioned above is a process in itself and is carried out in several steps which are not fully detailed. Automation of facial reconstruction follows a similar path.

1. A digital 3D model skull is created using 3D scanners.
2. The 3D skull model needs to be properly oriented in order to extract the skull measurements and the features automatically.
3. Based on the skull measurements obtained, the age, gender and race information can be extracted from the database.
4. Landmarks on the skull must be located automatically. This is possible only if the orientation and features of the skull are known from step 2.
5. Virtual dowels are automatically placed on the landmarks.
6. A face is built over the skull equipped with landmarks.

A lot of work has been done in this field. A wide range of 3D scanning systems are available for obtaining the 3D model of skull. Once the skull measurements are obtained, the age, gender and race information can be easily retrieved. Techniques have also been developed for automatic placement of landmarks in the case where skull orientation is known, and a variety of techniques are already developed for reconstructing a face model over a skull equipped with landmarks. One of the unanswered problems addressed in this thesis is the automatic orientation of the 3D skull model which is.

Here is a brief overview of rest of the thesis starting with problem formulation followed by issues discussed in each chapter.
Chapter 2 deals with some basic mathematical background required to understand the algorithms developed in this research. The 3D scanning system used in this thesis is discussed in Chapter 3. A NURBS surface skull model is used as a generic skull model for this work. The model description, its advantages and limitations are summarized in Chapter 4. In Chapter 5 a new algorithm is introduced for automatic registration of 3D models which has fewer limitations when compared to existing registration algorithms. Several deformation methods exist for fitting models to point cloud data. An alternative deformation method for fitting a NURBS model to 3D point cloud is discussed in Chapter 6. Quantitative measurements of the registration and surface fit are discussed in Chapter 7. Limitations and future work on this project are discussed in Chapter 8.
Chapter 2. Mathematical Background and Literature

Survey

The history of forensic facial reconstruction goes back to the 18th century. [Grüner, 1993] and the process of forensic facial reconstruction is constantly evolving. Some of the traditional facial reconstruction techniques are briefly discussed in this chapter. With the advent of high speed computing, it is now possible to computerize parts of this process. Several attempts of computerized forensic facial reconstruction have been made and are summarized in the later section of this chapter. However complete automation of the process remains a challenge.

Basic mathematics used and the skull anatomy nomenclature required for the understanding of the algorithm are summarized first, followed by review of manual and computerized forensic facial reconstruction. The chapter ends with a discussion work related to this thesis.

2.1 Mathematical Background

This section covers elementary mathematics used in computer aided geometric design and applied in this thesis. Notations used for vectors, matrices and other elements are introduced and a brief discussion included for coordinate systems and axes, Euclidian transformations, affine transformations, singular value decomposition, NURBS surfaces and their properties.

Vectors and matrices are the building blocks of all the affine transformations, euclidian transformations and singular value decomposition. Vectors will be represented by small, bold face letters $\mathbf{v}$. Transformations will be indicated with braces as follows $[T]$. Data points will be denoted by the bold italics
letter “D” where as model points will be represented as a bold Italicics “M”. Three
dimensional point coordinates will be represented by capitol letters, e.g. P.

2.1.1 Coordinate Axes Transformations

A coordinate system is a frame of reference for locating points in a space
of dimension n Coordinate axes in three dimension are a set of three orthogonal
unit vectors at the origin of the coordinate system.

A common operation required in most computer aided design applications
is the transformation of coordinate systems which can be interpreted in either of
two different ways. First, it can be viewed as an actual mapping of one point set
into another, i.e. moving the data relative to a fixed frame of reference. Second,
the same transformation can be interpreted as representing all the data
coordinates in a different frame of reference.

There are four fundamental transformations used in geometric modeling
and computer aided design namely translation, scaling, rotation and skewing
[Hill, 2001]. These transformations can be applied individually or in combination
to achieve a large number of desired effects. A transformation consisting of any
combination of the above mappings is called an affine transformation. Affine
transformations exhibits the ratio preserving property, i.e. straight lines are
mapped into straight lines such that the ratio of distance between any three
points along the line is unaltered [Mortenson, 1997].

Three dimensional transformations can be represented in a 4 X 4 matrix
form which is a compact and efficient scheme for computer implementation. As
shown in equation 2.2, a 3D point with coordinates (x, y, and z) is described by a
four element row vector. The ‘1’ is called the homogenous coordinate and is explained later in this section.

\[ P' = P[T] \]  \hspace{1cm} (2.1)

\[
\begin{bmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{bmatrix} =
\begin{bmatrix}
  a & e & i & 0 \\
  b & f & j & 0 \\
  c & g & k & 0 \\
  d & h & l & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix}
\]  \hspace{1cm} (2.2)

The components responsible for translation along the three principal directions are \(d, h\) and \(l\). Elements \(a, f\) and \(k\) in the transformation matrix incorporate scaling in \(X\)-, \(Y\)- and \(Z\)- directions respectively. Rotations about principal axes involve 4 elements. [\(X\)-axis: \(f, j, g\) and \(k\)], [\(Y\)-axis: \(a, i, c\) and \(k\)] and [\(Z\)-axis: \(a, e, b\) and \(f\)] with the direction of positive rotation is defined by right hand rule [Thompson, 1992]. Elements \(e, i, b, j, c\) and \(j\) in the matrix in equation 2.2 incorporate the skewing action.

The fourth row vector component in \(P = [x \ y \ z \ 1]\), the constant one (1), is the homogeneous coordinate. There are various advantages to using the homogeneous coordinate such as allowing translations to be included in matrix formulation and the square matrix helps in computing composite transformations through matrix multiplication. It also facilitates matrix inversion and global scaling.

One of the problems to be addressed in this research is to find a composite transformation in the case where, the initial state of the model and final state of a model are known. It is a linear system problem,

\[ M [X] = D \]  \hspace{1cm} (2.3)
where:

\( M \) and \( D \) are the subsets of the model and dataset respectively;

\( X \) is 4 X 4 transformation matrix which aligns the model \( M \) with the data \( D \);

\( M \) is a \( N \times 4 \) matrix, where \( N \) is the number of points in the model;

\( D \) is a \( M \times 4 \) matrix, where \( M \) is the number of points in the dataset.

\[
[X] = M^{-1}D
\] (2.4)

The transformation \([X]\) can be computed by multiplying the inverse of \( M \) with \( D \). However, \( M \) and \( D \) are not necessarily square matrices and hence not directly invertible. A matrix decomposition technique, Singular Value Decomposition (SVD) [Gentle, 1998] is therefore used to find a pseudo-inverse of this matrix. Using this technique, singular elements of a matrix are isolated from non-singular elements using equation 2.5. “Singular elements” are then inverted and combined with non-singular elements to compute a pseudo-inverse of non-singular matrix. With SVD, the model data \( M \) is decomposed into 3 separate matrices \( U \), \( S \) and \( V \)

\[
[M] = [U][S][V]
\] (2.5)

where:

\( U \) is \( N \times N \) orthogonal matrix;

\( V \) is \( 4 \times 4 \) orthogonal matrix and;

\( S \) is \( N \times 4 \) is diagonal matrix of singular values, consisting of positive numbers or zeros.

A pseudo-inverse of \( S \) can be computed directly by replacing the diagonal elements with their respective reciprocal as shown in equation 2.6.
The pseudo-inverse of $M$ can then be computed using equation 2.7.

$$[M^+] = [V][S^+][U^T]$$  \hspace{1cm} (2.7)

From this, the transformation $[X]$ is computed using equation 2.8.

$$[X] = [M^+][D]$$  \hspace{1cm} (2.8)

A euclidian mapping $[X]$ is used for initial registration of the 3D model to the scanned data and is discussed in detail in Chapter 5. Affine transformations obtained by SVD techniques are then used to more closely fit the model to the data as described in Chapter 6.

**2.1.2 Non Uniform Rational B-Spline (NURBS) Surface**

The model used for this research is a NURBS surface skull model and is detailed in Chapter 4. This section provides background on the formulation.

NURBS is an acronym for Non Uniform Rational B-Spline [Piegl, 1997]. NURBS is an industry standard, parametric polynomial spline representation for 3D geometries. Geometric entities such as lines circles, ellipses, spheres, tori and other free form surface geometries such as human body parts and car parts can be represented very accurately using NURBS models. The NURBS representation of complex geometry is typically a more compact form when compared to the information required by faceted and voxel geometry [ROGERS, 1991]. The NURBS algorithm can be implemented accurately and efficiently on a computer. NURBS curve and surfaces exhibit similar behavior.
A NURBS curve or surface is defined by four entities: its polynomial degree, a set of control points, knot vectors and an evaluation rule. The degree is a positive integer essentially setting the polynomial degree. The control points are an ordered list or an array of 3D points (X, Y and Z coordinates), each with an associated weight. When the weights of control points vary, the surface is called rational, ‘R’ in the NURBS acronym. Knot vector parameter values essentially correspond to the relative spacing of control points in the parametric domain. The evaluation rule is a mathematical formula incorporating the control points, knot vectors and weights.

The BS in NURBS acronym stands for B-Splines which are polynomial basis functions determined by the knots values. The B-Spline basis functions are computed for a particular input parameter pair (U, V) value which determines how the control points and weights are blended to obtain a point on the 3D surface. The governing equation for NURBS [Piegl, 1997],

The knot vectors are lists of numbers corresponding to the parametric domain. The length of the knot vector in a parametric direction is \((\text{degree} + N - 1)\),
where $N$ is the number of control points in that direction. *(N-degree)* is equal to the number of curve spans or segments combined to form a spline curve.

\[
S(u,v) = \frac{\sum_{i=0}^{m} \sum_{j=0}^{n} N_{i,p}(U)N_{j,q}(V)w_{i,j}P_{i,j}}{\sum_{i=0}^{m} \sum_{j=0}^{n} N_{i,p}(U)N_{j,q}(V)w_{i,j}}
\]  

(2.9)

where:

- $U$ and $V$ are knot vectors as follows;
  
  \[
  U = \{u_0,...,u_0,u_1,...,u_{i-1},u_i,...,u_l\};
  \]

  \[
  V = \{v_0,...,v_0,v_1,...,v_{j-1},v_j,...,v_l\};
  \]

- $N_{i,p}$ and $N_{j,q}$ are B-spline basis functions;

- $P_{i,j}$ are the control points;

- $w_{i,j}$ are the weights on the control points;

- $n$ is the number of control points in $v$- parametric direction;

- $m$ is the number of control points in $u$- parametric direction.

Several conditions must be satisfied by the knot vectors. The values increase monotonically down the list. Any knot value can be repeated, known as its *multiplicity*, but no more times than the degree of the NURBS. A knot value repeated *degree* many times is said to have a *full multiplicity*. The knots are called uniform if the values are equally spaced. Unequal spacing of knot values leads to the “non-uniform” or NU in the NURBS acronym.
2.2 Skull Anatomy

This section deals with the terminology of the mandible and skull bones used in this thesis [Berkovitz, 1988]. The mandible is a U-shaped bone forming the lower jaw, articulating with the temporal bone on either side of the skull. Figure 2.2 captures different sections of the mandible which are labeled from 1 through 11 followed by a brief paraphrasing of Strauss’s description for each bone feature.

http://www.bio.psu.edu/faculty/strauss/anatomy/skel/mandible2.htm

Figure 2.2 Strauss’s list of mandible features

1. **Mandibular Condyle**: An eminence at the end of the ramus of the mandible that articulates with the skull.

2. **Mandibular Notch**: A small indentation between mandibular condyle and Coronoid Process.

3. **Coronoid Process**: The triangular anterior projection of the mandibular ramus.

4. **Mandibular Ramus**: A projected part of the mandible.
5. **Mandibular Angle:** An angle formed by the junction of the posterior border of ramus and inferior body of mandible.

6. **Oblique line:** The line on the external surface of the mandible that extends from the mental tubercle to the ramus and separates the alveolar and basilar parts of the bone.

7. **Body:** The heavy, U-shaped, horizontal portion of the mandible extending posteriorly to the angle where it is continuous with the ramus. It supports the lower teeth.

8. **Alveolar Process:** The ridge on the surface of the upper and lower jaws containing the tooth sockets.

9. **Mental Foramen:** The front of the opening of the mandibular canal on the body of the mandible alongside and above the tubercle of the chin.

10. **Mylohyoid Line:** A ridge on the inner side of the bone of the lower jaw extending from the junction of the two halves of the bone in front to the last molar on each side and giving attachment to the mylohyoid muscle and to the superior constrictor of the pharynx.

11. **Mandibular Foramen:** The opening on the medial surface of the ramus that leads into mandibular canal and transmits blood vessels and nerves supporting the lower teeth.

   Figure 2.3 depicts bones which are important for skull alignment and subsequent fitting of a model to the data.
1. **Frontal Bone**: A cranial bone consisting of a vertical portion corresponding to the forehead and a horizontal portion that forms the roofs of the orbital and nasal cavities.

2. **Maxilla Bone**: A pair of bones of the skull fusing in the midline and forming the upper jaw.

3. **Zygomatic Bone**: A quadrilateral bone that forms the cheek prominence and articulates with the frontal, sphenoid, temporal, and maxillary bones.

4. **Nasal Bone**: An elongated rectangular bone that forms the bridge of the nose.

5. **Orbital Cavity**: The bony cavity containing the eyeball and its associated muscles, blood vessels and nerves.

6. **Nasal Septum**: The wall dividing the nasal cavity in two halves.

7. **Parietal bone**: Either of two large irregularly quadrilateral bones between the frontal and occipital bone that together forms the side and roof of the skull.

http://www.gwc.maricopa.edu/class/bio201/skull/latskul.htm
Figure 2.3 Strauss’s list of skull features
8. *Occipital Bone*: A curved, trapezoid compound bone that forms the lower posterior part of the skull.


### 2.3 Forensic Facial Reconstruction

Human remains identification is a challenging field of forensic science. In cases where physical evidence is available, the remains may be linked to a known person who is missing from the local area. Medical records can also be used to identify the victim. However, in the absence of additional physical evidence and the medical records, forensic facial reconstruction is the only means of facilitating victim identification [Jones, 2001]. Current methods used for forensic facial reconstruction are discussed in this section.

#### 2.3.1 Age, Gender, and Race Determination

The most essential information required for facial reconstruction is the age, the gender and the race of the victim, which can be obtained from the skull characteristics. The age of an individual is indicated by the sutures and the dentition of the skull. The shapes of the ridge and the chin are useful in determining the gender of individuals and lastly the shape of the mandible is used to identify the race [Archer 1992].

#### 2.3.2 Types of Forensic Facial Reconstruction

A variety of facial reproduction techniques exist. Most popular among them are superimposition [Krogman, 1986], craniofacial reconstruction in 2D [Iscan, 1993], and 3D [Manhein, 2002]. Amongst other methods for a less
decomposed body are facial restorations which involve rebuilding of damaged facial tissues which are attached to the skull [Archer 1992].

2.3.2.1 Superimposition

A photograph of an individual is manually superimposed on the picture of a skull for comparison. Most often this technique is used to trivially eliminate victims who are not potential matches [Krogman, 1986].

Photographs of possible victims are first collected. Then photographs of the skulls are taken at a particular orientation closely matching that in each individual’s photograph. Both the photographs of the skull and the individual are enlarged to life size. The skull photograph is then superimposed on the individual’s photograph. These steps are repeated for each individual and the images are compared for likeliness.

The major limitations of this method are that, a reference is required for enlargement of pictures of the skull and an individual’s photograph, also the skull needs to be positioned such that it matches the orientation of the individuals photograph [Archer 1992].

2.3.2.2 2D Craniofacial Reconstruction

This 2D craniofacial reconstruction technique is more reliable than superimposition [Iscan, 1993], and consists of the following steps. Tissue markers are glued on the skull at the proper pre-determined landmarks. The skull is then positioned so that photographs of its front and side views of skull are taken. Using a reference ruler in each, all images are printed true to scale. This is
done by comparing the actual length ruler to the ruler reference in each photograph.

The front view and the side view photos of the skull are then taped on two separate wood boards in the horizontal position [Taylor, 2001] and then the two are placed next to each other. The photographs are then covered by transparent vellum sheets.

Using the tissue markers as guidelines, a sketch is drawn along the contours of the skull. The mouth, nose and eyes are sketched using the same measurements as used in 3D craniofacial reconstruction. Hair characteristics are approximated based on whatever samples are found on the scene. In case of lack of any physical evidence, a victim’s age gender and race information is used to approximate the hair style.

2.3.2.3 3D Clay Facial Reconstruction

3D clay facial reconstruction is a blend of art and science. It is an old technique which is still in use and is constantly evolving [Manhein, 2002]. The basic process is summarized below.

Any recovered skull is first cleaned so that all the tissue and skin remains are removed. The skull is then placed on a clay ring. The mandible is properly positioned and firmly glued to the skull. Holes on the skull (if any) are taped. The skull is then placed in the Frankfort horizontal position [Taylor, 2001]. Eye orbits, nasal aperture and auditory meatus are gently stuffed with cotton. The cotton is then covered with masking tape and a layer of clay is applied.
Based upon the various skull measurements, a victim’s age, gender and race are identified and corresponding tissue depth data are chosen. Markers are then cut and glued on corresponding skull features. Eye balls are carefully placed in eye sockets and are supported by filling and surrounding each with clay. 4 mm thick and approximately 13 mm wide strips are glued between the markers. Double strips are placed where tissue depth is greater than 4 mm. Additional clay is filled in between the grid formed by the strips.

The next task is to build the nose and mouth. Using the measurements recorded in the case record, nose width and projections are calculated, and used to build a nose. Clay is then artistically placed over teeth and positioned between the gum lines. Lips are shaped based upon the age, gender and race information. The surface is now smoothed according to the underlying facial muscles. For a more detailed discussion of facial reconstruction refer to [Manhein, 2000].

2.3.3 Clay Facial Reconstruction Issues and the Need for Automation

As described in section 2.3.2 the traditional clay facial reconstruction method requires expertise in the field of anatomy, forensic science and art. The process is very time consuming and when a victim is not identified, a second face reconstruction may be attempted with slightly different features. Since the original skull is needed, the first reconstructed face needs to be taken apart in order to restart the process. There are no exact rules for the reconstruction which makes computerization of the process more challenging.
The requirement of expert knowledge, the time required and the challenging nature of the problem has motivated a number of research attempts to computerize the forensic facial reconstruction process. A lot of work has already been done in computerizing parts of the process, which is discussed in next section of this chapter. However, complete automation of the facial reconstruction process remains a challenge.

2.4 Previous and Related Work on Computerized Forensic Facial Reconstruction

The process of forensic facial reconstruction has no exact rules. This leads to a variety of ways of addressing this problem. Various researchers have worked on the problem and have provided completely different solutions. Mark W. Jones [Jones, 2001] uses volumetric data and cross correlation techniques for the process. Use of extensive statistical tools such as Generalized Procrustes Analysis (GPA) and Principal Components Analysis (PCA) is proposed by Matthew Cairns [Cairns, 2002]. Another approach to computerizing the process is presented by Katrina Archer [Archer, 1997] in which she attempts to closely simulate the manual facial reconstruction process. A physics based head model which includes skin surfaces, virtual muscles, a mass spring system and landmarks is used by Kähler for the facial reconstruction [Kähler, 2003]. Each of these is reviewed in sections to follow, their advantages and limitations are also discussed.

2.4.1 Facial Reconstruction Using Volumetric Data

Mark Jones [Jones, 2001] proposed a method of facial features reconstruction from discovered remains by comparing volume data of the
remains with that of a reference head. A CT scanner is used to generate a volumetric model of the discovered skull. A reference skull having the same age, gender and race characteristics is chosen. Using correlation techniques, a correspondence is created between the discovered skull and the reference head. The soft tissue from the reference head is then mapped onto the discovered skull using this correspondence.

Jones proposes a mathematical morphing algorithm which uses distance field calculations for the computation of corresponding points [Jones, 2001]. This is very much a work in progress and a complete facial reconstruction has not yet been tested. The correlation is based upon only one view, in the direction of viewing angle. This leads to errors in reconstruction of features such as ears. Also, it is assumed that the images of two skulls are pre-registered.

2.4.2 3D Facial Modeling and Forensic Facial Reconstruction

Cairns’s work [Cairns, 2002] is also an ongoing research project. A large database of 3D volumetric faces is created. Each face model has roughly same number of points and in the same relative position with respect to particular facial features. The same triangulation is used to render each face. Using Generalized Procrustes Analysis, a one-to-one correspondence is created between all the faces [Cairns, 2002].

The process of creating a correspondence is as follows. A VRML version of the C3D (a CAD file format) face is loaded into a java class, called MESH, created by Matthew Cairns. Landmarks are interactively placed on the surface of the face. These are anatomical landmarks used for facial reconstruction and
intuitively chosen geometric landmarks which are useful in defining the facial features. These landmarks are then triangulated in a predetermined order [Cairns, 2002].

The mesh is then refined by joining the midpoints of each triangle. Points are clipped to the surface and the process is repeated until the desired smoothness is achieved. A database of 50 faces is created. A standardized mesh as described is fit to each face. A set of faces is chosen based upon the age, gender and the race of the skull that needs to be identified. These faces are combined linearly using Principal Component Analysis to approximate the victims face. Using barycentric coordinates, VRML textures are also interpolated [Cairns, 2002].

For more detailed description of Generalized Procrustes Analysis and Principle Component Analysis refer to Matthew Cairns 3rd year report [Cairns, 2002]. As mentioned earlier, this work is still in progress and results are yet awaited. The proposed method requires a lot of human interaction and also computer and mathematical skills.

2.4.3 Craniofacial Reconstruction Using Hierarchical B-Spline Interpolation

Archer [Archer, 1997] presents the first comprehensive computerized 3D facial reconstruction system. The process closely simulates the manual facial reconstruction process.

First, the skull is digitized using any 3D scanning system. Anatomical landmarks are located and virtual skin depth dowels are interactively placed at the proper locations and oriented by the forensic artist. Once the dowels are placed,
a generic B-Spline surface model of a head is interactively placed around the skull. A hierarchical B-Spline surface can be edited interactively and hence suits the purpose. The surface model is then interactively fit to the virtual dowels placed on the surface of the skull. Finally, the surface is smoothly and evenly interpolated through these dowels to obtain a candidate face for the victim. The orientation and location of the dowels can be altered interactively, and for the same skull, multiple faces can be generated fairly quickly [Archer, 1997].

Although it is a completely computerized 3D facial reconstruction tool, it requires a lot of interactive input. Additional computer modeling skills are required in order to create a template head model for the skull in consideration. Also the interpolation process is tricky and requires a certain level of skill to create a model which is good enough for the process.

2.4.4 Expressive Faces from Skull Data

The system proposed by Kähler [Kähler, 2003] has many advantages over the traditional and computerized techniques discussed earlier in this chapter. Once the skull is digitized, it takes only about an hour for complete facial reconstruction and the face model can be altered very easily. Expressive faces are generated utilizing a virtual muscle layer underneath the skin layer on a deformable, anatomy based head model [Kähler, 2003]. The muscle and skin layers are deformed to fit a given skull data.

3D skull data is acquired by either volume scans or by range scanning. Using mesh decimation techniques [Garland, 1997], a triangular mesh model consisting of 50-250k polygons per skull is produced. Landmarks are interactively
placed on the surface of the skull. Initially, landmarks are oriented normal to the surface. Each landmark is automatically scaled to the local tissue thickness. The location and orientation of landmarks can be altered for refinement. Once the landmarks are placed, the deformable head model can be fit onto the skull [Kähler, 2003].

A physics-based head model is used for this purpose. It consists of the skin surface, virtual muscles, a spring mass system and landmarks. The skin surface is a triangular mesh consisting of 8164 triangles. Virtual muscles are used to control various facial expressions. The model consists of 25 muscles, each an array of fibers which can contract in linear or circular fashion. The muscle shape is computed automatically to fit underneath the skin. A spring-mass system is added after the face is fit to the skull. The spring mass system connects the skin and the muscles to the skull. Most landmarks on the skin surface correspond to the skull landmarks. For final fitting of the head model, a few extra landmarks are used which are defined only on the surface of the skin. For a more detailed discussion of the fitting process refer to [Kähler, 2003].

2.5 Related Model Registration and Fitting Algorithms

The previous section summarizes various computerized methods for facial reconstruction. Amongst these, Archer’s work and Kähler’s work are major milestones towards the automation of the process. However, complete automation remains a challenge. One of the key tasks to be addressed for complete automation is automatic detection of landmark locations and placement
of landmarks on the skull. This requires automatic recognition of skull features
which can be a computationally expensive proposition.

An intuitive way to achieve the goal is to fit a skull model, already
equipped with landmarks and with known orientation and position, to the scanned
skull data. This involves automatic registration and then controlled deformation of
the generic skull model to fit the scanned skull. An automated, feature based
data fitting technique is presented by Gregory Dobson [Dobson, 1995], for fitting
of 2D facial profiles and 3D foot models. This technique is summarized in the
next section of this chapter.

A generic model of the skull is required for this purpose. The basic idea is
to deform the generic model so that it fits the point cloud data with a necessary
tolerance. Since the model is to be deformed, a robust model is required such
that it should get deformed to fit the data without losing inherent features of the
model. As such various surface models, their formulation, advantages and
limitation are discussed in this section.

Once a suitable surface model is chosen, registration with the scanned
data is required. Several techniques exist for registration of 3D models and are
discussed in this section. Most of them require user input. Others have ambiguity
issues with symmetric models.

Various techniques also exist for surface fitting. A few of these techniques
along with their advantages and limitations are reviewed in the following sections
2.5.1 Generic Surface Model Selection

Triangulated surface models are the most commonly used 3D models in visualization [Kobbelt, 2000]. A high density surface model is required to create a smooth surface which results in higher computation time and larger memory requirements.

Since triangular mesh data is typically unstructured and highly complex, model deformations are difficult to control. As such, a number of researchers have tried to simplify surface triangulation. Leif P. Kobbelt [Kobbelt, 2000] introduced an interactive approach to point cloud triangulation which is able to process highly complex and completely unstructured hybrid data. Enhanced graphics hardware features such as a raster manager and the z-buffer for specific tasks in overall procedure are used for the purpose. It is a good technique to generate 3D models, although, the models created are not amenable to further deformation. Other techniques such as local triangulation [Linsen, 2001] and surface reconstruction through geometric data fusion [Garcia, 1994] are available to create smooth 3D models but still suffer the same problem.

A more robust model is required for the purpose and a NURBS surface model is well suited. The NURBS surface gives more freedom and flexibility to represent complex and free from shapes and is much more compact form. NURBS surfaces are governed by control points and 2D parametric knot vectors.

2.5.2 Registration of 3D Models

This task has been explored by several of researchers. Most of the existing methods either need user interaction or have limitations over the type of
models to be used. Amongst these, some methods are discussed below.

2.5.2.1 A Method of Registration of 3-D Shapes

A representation independent method for accurate registration of 3D shapes is presented by Besl [Besl, 1992]. This method is based on the Iterative Closest Point (ICP) algorithm. Closest point associations from model to data are obtained and the model is transformed based upon these associations.

The steps involved in the algorithm are as follows.

1. The closest point associations between two point sets are obtained using ICP.
2. Based upon these associations, the registration vectors and the associated cost are computed.
3. The registration vectors are applied to translate the dataset.
4. Steps 1 through 3 are repeated until the mean square error is larger than a preset threshold.

This method is computationally expensive for registration since it uses ICP also initial rotation and translation guesses are required and no mention is made of method being automatic and able to align symmetric models.

2.5.2.2 Robust Euclidean Alignment of 3D Points Sets.

Geometric alignment of two roughly pre-registered, partially overlapping 3D models is discussed by Chetverikov [Chetverikov, 2002]. A trimmed Interactive Closest Point (TrICP) algorithm is used for the purpose. This algorithm uses Least Trimmed Square (LTS) instead of Least Median Square (LMedS) because it has a smoother objective function and better convergence
rate. Unlike LMedS which minimizes the median of sorted sequence, in LTS, after sorting the square of errors, a certain number of values are minimized.

It is assumed that the two models are roughly pre-registered and the overlap between the models should be more than 50%. Also the shape of the overlap must be characteristic enough to allow unambiguous matching [Chetverikov, 2002] of major features. The iterative closest point (ICP) algorithm also overlooks the fact that all the data-points are not paired correctly with the points in the model.

This method of alignment is better than the conventional ICP approach; however, it is not well suited for the initial alignment since it requires roughly preregistered models with an overlap such that the overlap represents characteristic shape in order to avoid ambiguous shape matching.

2.5.2.3 3D Model Alignment and Retrieval System

A new concept of 3D model alignment in presented in Chen's paper [Chen B. Y., 2002]. The basic premise is that two similar 3D models will look the same from a common viewing direction. The model to be aligned is first translated and scaled to the second. The translation is based upon matching the mid points of corresponding models. The scaling correlates the maximum and minimum limits of two models.

2D profiles of both the models are captured referencing vertices of a dodecahedron as viewing points. According to Huber and Hubert [Huber, 2001], 20 viewing angles can vaguely represent the shape of a 3D model. These 20, 2D
shapes contain enough information about the 3D model and are used as bases for alignment of 3D models [Chen, 2002].

All 2D shapes are rendered from camera sets for both the models. OpenGL [Chen, 2002] is used to render 2D silhouettes. A virtual camera is placed at each vertex of dodecahedron facing towards center point. Features are extracted from each 2D shape; however no mention is made of it being done automatically or manually. Each 2D shape from the first model is then matched with each 2D shape in the second model. A region-based shape descriptor, MPEG-7, is used to measure the similarity between 2D silhouettes. The pair with minimum error is considered for obtaining a single rotation matrix which is applied to the first model. The process is then repeated for finer rotation angles.

2.5.2.4 Surface Registration by Matching Oriented Points

Registration of 3D surfaces acquired from different view points is discussed by Johnson [Johnson 1997]. The method aligns two views of the same model or scene. First, using a range sensor, 3D data of a scene is acquired from two views and 3D data is converted into triangular meshes. A new method for point correspondence is presented, for the matching points on the surfaces of the objects. Based on the set of geometrically consistent point correspondences, a rigid Euclidian transformation is computed which aligns the two views.

Johnson’s method of point correspondence is based on the idea of a spin-image. A unique spin-map $S_O$ is associated with each distinct oriented surface point $O$. A Spin map is a 2D basis $(p, n)$ where, $p$ is position vector and $n$ is surface normal at that point. A 2D set of points, called a spin-image $I_{(O,M)}$, is
created by applying $S_o$ to all the points on the surface $M$ associated to the oriented point $O$. The spin-images created for each surface are then compared and a correspondence is established between the oriented points. Spin images are saved as 2D arrays of floating point numbers. Correlation between two images $P$ and $Q$ is then computed using the function $C$,

$$
C(P, Q) = (\tanh(R(p, q)))^2 - \lambda \left( \frac{1}{N-3} \right)
$$

(2.10)

where:

- $R$ is a function to compute number of pixels;
- $\lambda$ is expected variance in correlation coefficient;
- $N$ is number of overlapping bins.

A high value of $C$ indicates that the images are highly correlated. To avoid uncertainties due to noisy data, scene symmetry and inherent symmetry, multiple symmetry are established between the model and a scene. A transformation from model to scene is computed for each correlation. The results are verified by an iterative closest point approach.

This method is ideal for registration of different views of the same 3D surfaces model. However, alignment of 2 similar models is not discussed in this paper.

### 2.5.2.5 Feature Based Models for Anatomical Data Fitting

A combination of various 3D methods is used for alignment of feature based foot model to scanned data in Dobson [Dobson, 1995]. The center of mass and principal axes for the data and the model are computed. The model is positioned to the data by collocating the model and the data’s centers of mass.
The principal axes of the model and data are then aligned. The model is then scaled independently along each direction based on a max/min box enclosure.

The position and orientation of the model are then refined by performing a least square optimization and minimizing the squared distances between the data and the model. This method is best suited for automatic alignment of model to data. However, since principal axes are used for the alignment, ambiguity issues arise in the case of symmetric objects and a more robust method needs to be developed which can handle symmetric as well as non-symmetric models.

2.5.3 Local Fitting of Model to Dataset

Skull shape varies with age, gender and race. Therefore alignment only of a model to a dataset is insufficient to detect features. The model must be deformed in order to fit it to the dataset within a permissible tolerance required by the forensic analyst.

A feature based deformation technique is presented by Dobson [Dobson, 1995]. As discussed in earlier section of this chapter a B-Spline model is first aligned and scaled to fit the data. Then, an iterative feature based model deformation is applied to refine the fit.

First a region of influence or a parametric neighborhood is defined for each feature in the model. Data points whose closest model point fall within the domain of influence of a particular feature are then used to estimate the deformation necessary for that particular feature.

Dobson indicates a characteristic or “feature point” for each pertinent model characteristics. An average error vector is calculated for each feature point
which indicates the approximate distance and direction in which that feature point must be moved in order to bring model closer to matching the data set. The feature point error vector is computed by averaging the error between each associated data point and its closest point on the model.

Feature points are related to the control vertices and hence a change in feature points correlates with the change in control vertex positions. Note that the number of feature points is generally less than the number of control vertices and hence a Singular Value Decomposition technique is used to resolve the necessary control vertex modifications. The residual error estimate and vertex deformation are repeated until one of two satisfying conditions is reached. First, the sum of squared data point distance error converges to an acceptable limit or second, a limiting number of iteration is reached. Control vertex and weight optimization techniques are then used to locally improve the surface/data fit.

This method is well-suited for fitting a B-Spline model to a point cloud. However, an alternative new method based on similar concept is investigated in this thesis which deforms the model by deforming the isoparametric contours formed by the control points.
Chapter 3. 3D Scanning Systems

Laser scanning systems are becoming popular for collecting 3D points from the surface of objects. A wide range of scanners are available for different scanning conditions and scanning of different types of models. Amongst them, the most popular are scanners from Cyberware, Roland and Polhemus. Cyberware and Roland scanners are primarily too bulky and expensive for the purpose of this work. A readily available Polhemus Cobra is the scanner used for this project. The scanning and processing procedure are discussed in this chapter.

3.1 FastSCAN Cobra

FastSCAN Cobra, (see Figure 3.1) a hand held scanner, from Polhemus suits the purpose of scanning the skull and mandible. This scanner is a non-contact type range finder based on projection and simultaneous detection of laser light, coupled with a transmitter which helps in finding the location and orientation of the range finder.

A range camera is used as a range finder and is contained in a hand held wand along with the laser light emitter. Tracking is done with a Polhemus FASTRACK magnetic tracker.

Scanning involves sweeping the wand smoothly over the surface of an object, in a manner similar to spray painting. Redundant 3D points are eliminated as overlapping sweeps are combined based upon the resolution and tolerance set by the user.
The 3D surface generated after processing can be exported to a range of industrial standard formats for loading into other applications. The file format required for this research is .MAT format, which is a Matlab® readable format.

The complete FastSCAN Cobra Handheld Laser Scanner System is shown in the Figure 3.1. The FastSCAN Cobra consists of a wand which is at the top in the Figure, the processor at the bottom, the cube like object is the transmitter, and the optional stylus at the top right.

![3D scanning system](www.polhemus.com)

Figure 3.1 3D scanning system. [FastSCAN®]

3.1.1 The Wand

The wand in Figure 3.2 (a) consists of a range camera and a laser line generator. The user projects the laser light on the object to be scanned and the camera records the profile generated by the intersection of the laser line and the
surface of the object. Triangulation is used to compute the 3D location of the profile with respect to the Wand.

The range camera is calibrated according to the position of wand and hence it must be handled with special care. The wand has two controls, the trigger and the sensitivity control, and four status indicators: power, laser, scan, and sensitivity level. The trigger is located on the underside of the handgrip, where as the sensitivity control and all other status indicators are located on the control panel on the side facing the user.

The trigger is a two position switch. The first is a preview position where the trigger is depressed only halfway and, the second scan position is when the trigger is fully pressed. With the trigger in preview position, the laser profiles are displayed in real time on the computer screen but nothing is recorded. This provides the operator with feedback which is useful in aligning the sweeps and searching to fill up the data gaps. With the trigger in scanning position, the laser profiles along with the surface recorded are displayed on the screen in real time.

Figure 3.2 (a) The wand and (b) Control panel of wand. [FastSCAN®]
The sensitivity of the laser is adjusted by two buttons on the control panel of the wand. The status of the sensitivity is displayed on the tapered bar like indicator as shown in the Figure 3.2(b). The use of sensitivity is discussed in detail at a later stage in this chapter.

3.1.2 Processing Unit

![Figure 3.3 The processor unit. [FastSCAN®] (a) Front view (b) Back view](a)

![Figure 3.3 The processor unit. [FastSCAN®] (a) Front view (b) Back view](b)

The processing unit consists of all the electronics for the FASTRACK magnetic tracker and the video processing. The processor unit and all the connections are shown in the Figure 3.3

3.1.3 Receiver

There are two receivers, one on the wand and the other is external receiver. The one on the wand allows the computer to determine the position and
location of the wand with respect to the transmitter. The external receiver is used to scan objects which are moving. The external receiver is attached to the model. The computer can detect the location and the orientation of the model with respect to the model and hence scanning of moving objects is possible. Figure 3.4 shows a typical receiver.

![Receiver Image](www.polhemus.com)

**Figure 3.4 The receiver[FastSCAN®]**

### 3.1.4 Transmitter

A transmitter is shown in Figure 3.5. 3D magnetic field is generated by the transmitter. The field amplitude determines the position and orientation of the receiver. Since the transmitter generates a magnetic field it should be placed away from metallic objects. Also the transmitter is required to be placed at least 1 m away from the CPU and processor, to avoid distortion in the magnetic field.

### 3.2 Ideal Scanning Environment

Any area with subdued lighting, no sunlight and no light from the outside source, is an ideal scanning environment. Scanning errors may arise if the camera views direct sunlight or a broad-spectrum light.
The processor unit, computer, and all other metal objects must be placed at least 1 meter away from the scanning area. The objects to be scanned should be placed on a wooden surface for best results with the transmitter placed as close as possible to the object being scanned. The wand should be held at between 100 mm -150 mm from the surface of the object for best scanning results. The resolution of the triangulation depends on the angle between the laser and the camera and reduces with increased wand to object distance. The ideal distance between the wand and the transmitter is 750 mm. The accuracy of the transmitter depends upon the strength of magnetic field which reduces with increased distance between transmitter and receiver.

### 3.3 System Requirements

3D scanning is a computationally expensive process and demands an optimum computer specification. The following list indicates the minimum system requirements.
• Microsoft Windows NT 4 with Service Pack 3 or above or Microsoft Windows 2000 with Service Pack 1 or above.
• Intel Pentium III 500 MHz equivalent or better.
• 256 MB RAM.
• ISA parallel port supporting ECP mode with DMA.
• OpenGL hardware accelerated graphics adapter in 32-bit true color model.

A higher configuration improves scanning and processing speed. An Intel Pentium IV processor with 512 MB RAM is used for this research.

3.4 Skull Preparation

The skull must be dry to avoid a semi-translucent sheen which causes scanning errors. If the skull is not dry, powder can be applied in order to make the surface more matted. For efficient scanning, the mandible and the skull model are scanned separately. The skull or mandible is placed on a clay mount so be held completely stationary and until the scanning is completed. The transmitter is placed at the bottom and special care is taken to make sure that it will not be disturbed during the scanning process. All the components must be connected to the processor unit, either being used or not. The scanner is turned on and the FastSCAN program is started.

3.5 The Scanning Process

The scanning process is carried out in several steps. First the scanner is initialized by pointing the laser towards the transmitter. The laser is then projected on the model. The profile is previewed in camera mode to make sure that the object is visible to the range camera. Once the scanner is initialized, the
sensitivity of the scanner is adjusted to accommodate the lighting conditions, surface types and colors, and scanning environment. The sensitivity control is located on the left side of the wand control panel. Level 1 is the least sensitive whereas level 6 is the most sensitive. The sensitivity should be adjusted to the lowest level that yields an unbroken red laser line in preview mode.

Multiple sweeps of the wand are required to scan a complete 3D object. The wand should be moved smoothly in one direction in a sweeping motion with a slight overlap in the successive sweeps. Abrupt movements should be avoided as it causes redundancy and error in scans. Repetitive scanning of the same part of the surface should be avoided.

As noted earlier, the distance between the wand and the surface of the object dictates the accuracy of the measured point on the object surface. Therefore the wand should be as close as possible to the surface be close to the transmitter. Resolution of the raw surface also depends on the speed at which the wand is moved although the best and worst scanning resolution can be set using the scanner limit controls. A higher smoothness value smooths the surface but also removes minute surface details, a lower value, on the other hand retains the surface details at the cost of smoothness.

There are no fixed values to be set for best results. All the parameters depend on the scanning environment, the surface texture and color, speed of scanning, and require adjustment before each scan. At times, the parameters will require modification while the scanning is in progress.
3.6 Processing and Exporting

The scanned surface is just a large collection of points or raw data. Additional processing of this raw data minimizes scanning errors and removes redundant data from overlapping sweeps. The processing also standardizes the resolution of the model, simplifies the facets, removes outliers and limits the total number of the objects contained in the raw surface.

Once processed, the surface is exported in .MAT file format suitable for the Matlab® application. MAT files contain three arrays namely, points, facets, and normals. Points and normals are n X 3 matrices, where n is number of points. Facets are an m X 3 matrix of point indices where m is the number of triangular facets. The exported surface can now be opened in Matlab along with the NURBS skull model, which is discussed in the next chapter.
Chapter 4. NURBS Surface Skull Model and NURBS Toolbox for MATLAB and Interface

A NURBS surface skull model is used as a generic skull template. The mandible and skull body are modeled and managed separately. The model description, its advantages and limitations are discussed in this chapter. For efficient handling of the NURBS model, a data structure is created in Matlab and is discussed in this chapter. A graphical user interface is developed for this project which is described in brief in this chapter.

4.1 Model Description

The NURBS skull model, in Figure 4.1 was purchased from www.3DcadBrowser.com in an IGES format. Figure 4.1 is a Figure of complete skull model displayed in Matlab 7 using the NURBS toolbox, discuss later in this chapter. Figure 4.2 is an exploded view showing the patch composition of the skull model.

It is very difficult to model the complex geometry of an entire skull with a single NURBS spline; hence the model discussed in this chapter is actually a collection of 46 distinct spline surfaces as shown in Figure 4.2. Control points are stored in a matrix of 4D homogeneous coordinates of size m X n where, m and n is number of control points the two parametric directions. All of the surfaces have geometric continuity with adjacent patches, however there is no explicitly imposed derivative continuity imposed at the boundaries of these models. This becomes important in deforming the model.
Figure 4.1 NURBS skull model
Figure 4.2 An exploded view of NURBS skull patches
4.1.1 Mandible and Lower Teeth

The mandible control mesh is a 24 X 61 matrix of 3D homogeneous control points. Figure 4.3 (a) and (b) illustrate isoparametric curves and the NURBS model respectively. Its control mesh is depicted as 61 isoparametric curves, each consisting of 24 control points. The corresponding knot vector in U direction consists of 28 values whereas knot vector in V direction consists of 65 values.

Each tooth is represented by its own NURBS model having a control mesh of varying size from 8X9 to 12X18. The corresponding knot vectors vary accordingly. Figure 4.3(c) shows isoparametric contours for all the lower teeth.
4.1.2 Frontal Bone

![Diagram showing isoparametric curves and NURBS surface model of the frontal bone.](image)

Figure 4.4 Frontal bone (a) Isoparametric curves (b) NURBS surface model

The control mesh of frontal bone is a matrix of 3D homogeneous points of size 73 X 9. Figure 4.4 (a) and (b) depicts the control mesh contours and the NURBS surface model of frontal bone respectively. Knot vectors in the U and V directions consist of 77 and 13 knots respectively.

4.1.3 Left and Right Eye Orbit

The left and right eye orbits have similar geometric properties and, as such, the models are symmetric as reflected in their respective control meshes and the knots vectors. Figures 4.5 (a) and (b) show isoparametric curves and NURBS surface models of left eye orbit and Figure 4.6 (a) and (b) show the corresponding right eye orbit.

The control meshes of both left and right eye orbit are 3D homogeneous control point matrices of size 36 X 15. The corresponding knot vectors in U and V direction consist of 40 and 19 values respectively.
4.1.4 Nasal Bone

The control mesh of the nasal bone is a matrix of 3D homogeneous points of size 14 X 7 matrix. Figure 4.4 (a) and (b) shows the isoparametric curves and the NURBS surface model of the nasal bone respectively. Its control mesh is
made of 7 isoparametric curves and each curve consists of 14 points. Knot vectors in the U and V directions consist of 18 and 11 knots respectively.

Figure 4.7 Nasal bone (a) isoparametric curves (b) NURBS surface model

4.1.5 Nasal Cavity

The nasal cavity is connected to nasal bone, septum, anterior nasal spine, left and right maxilla. Figure 4.8 (a) and (b) show the isoparametric curves and the NURBS surface of the nasal cavity.

The control mesh of the nasal cavity model includes a matrix of 3D homogeneous points of size 29 X 10. Figure 4.4 (a) and (b) depicts the isoparametric curves and the NURBS surface model of the nasal cavity. Its control mesh is illustrated by 10 isoparametric curves and each curve consists of 29 points. Corresponding knot vectors in U and V direction consist of 33 and 14 knots respectively.
4.1.6 Septum

The septum is contained inside the nasal cavity. The control mesh of the septum is a matrix of 3D homogeneous points of size 5 X 9. Figure 4.9 (a) and (b) shows the isoparametric curves and the NURBS surface model of the septum respectively. Its control mesh is made of 9 isoparametric curves and each curve
consists of 5 points. Knot vectors in U and V consist of 7 and 13 knots respectively.

![Fig. 4.9 Septum (a) Isoparametric curves (b) NURBS surface model](image)

**4.1.7 Occipital**

The occipital lobe is adjacent to the frontal bone, left and right eye orbits and the temporal bone. The control mesh of occipital is a matrix of 3D homogeneous points of size $57 \times 6$. Figure 4.10 (a) and (b) shows the isoparametric curves and the NURBS surface model of the occipital respectively. Its control mesh is made of 6 isoparametric curves and each curve consists of 57 points. Knot vectors in U and V direction consist of 61 and 10 knots respectively.

![Fig. 4.10 Occipital (a) Isoparametric curves (b) NURBS surface model](image)
4.1.8 Anterior Nasal Spine

The anterior nasal spine is adjacent to the nasal cavity, and the left and right maxilla. The control mesh of anterior nasal spine is a matrix of 3D homogeneous points of size 9 X 12. Figure 4.11 (a) and (b) depicts the isoparametric curves and the NURBS surface model of the occipital respectively. The control mesh is made of 12 isoparametric curves and each curve consists of 9 points. Knot vectors in U and V direction consist of 13 and 16 knots respectively.

![Figure 4.11 Anterior nasal spine (a) Isoparametric curves (b) NURBS surface model](image)

4.1.9 Left and Right Maxilla

The left and right maxillas are adjacent to the nasal cavity and anterior nasal spine. The left maxilla is adjacent to the left zygomatic bone where as the right maxilla is adjacent to the right zygomatic bone. The control meshes of both left and right maxilla are a matrix of 3D homogeneous points of size 12 X 18.
Figure 4.12 (a) and (b) demonstrate the isoparametric curves and the NURBS surface model of the left maxilla and Figure 4.13 (a) and (b) show the isoparametric curves and NURBS surface of right maxilla respectively. Its control mesh is made of 18 isoparametric curves and each curve consists of 12 points. Knot vectors in U and V direction consist of 16 knots 22 knots respectively.

![Figure 4.12 Left maxilla (a) Isoparametric curves (b) NURBS surface model](image1)

![Figure 4.13 Right maxilla (a) Isoparametric curves (b) NURBS surface model](image2)

### 4.1.10 Left and Right Zygomatic Bone

The left zygomatic bone is adjacent to the left maxilla, left malar and the left eye orbit, while as right zygomatic bone is adjacent to the right maxilla, right
malar and the right eye orbit. The control meshes of both left and right zygomatic bones are a matrix of 3D homogeneous points of size 19 X 8. Figure 4.14 (a) and (b) represent the isoparametric curves and the NURBS surface model of the left zygomatic bone and Figure 4.15 (a) and (b) show the isoparametric curves and NURBS surface of right zygomatic bone respectively. Its control mesh is made of 8 isoparametric curves and each curve consists of 19 points. Knot vectors in U and V direction consist of 23 and 12 knots respectively.

Figure 4.14 Left zygomatic bone (a) Isoparametric curves (b) NURBS surface model

Figure 4.15 Right zygomatic bone (a) Isoparametric curves (b) NURBS surface model
4.1.11 Left and Right Malar

The left malar is adjacent to the left zygomatic bone and the occipital, whereas the right malar is adjacent to the right zygomatic bone and the occipital. The control meshes of both left and right malar are a matrix of 3D homogeneous points of size 20 X 14. Figure 4.16 (a) and (b) show the isoparametric curves and the NURBS surface model of the left malar and Figure 4.17 (a) and (b) illustrate the isoparametric curves and NURBS surface of right malar respectively. The control mesh is made of 14 isoparametric curves and each curve consists of 20 points. Knot vectors in U and V direction consist of 24 and 16 knots respectively.

(a)                                                                                     (b)

Figure 4.16 Left malar (a) Isoparametric curves  (b) NURBS surface model

(a)                                                                    (b)
Figure 4.17 Right malar (a) Isoparametric curves  (b) NURBS surface model
4.1.12 Temporal Bone

The temporal bone is adjacent to the occipital. The control mesh of temporal bone is a matrix of 3D homogeneous points of size 71 X 15. Figure 4.18 (a) and (b) describe the isoparametric curves and the NURBS surface model of the temporal bone respectively. The control mesh is made of 15 isoparametric curves and each curve consists of 71 points. Knot vectors in U and V direction consist of 174 and 19 knots respectively.

![Figure 4.18 Temporal bone (a) Isoparametric curves (b) NURBS surface model](image)

The mandible model is managed individually while other models except the septum are grouped together to form a skull body. The septum is not used for deformation purposes since there is no corresponding data in the scanned
dataset. Subgroups of models in the skull body are formed for local deformation which is discussed in Chapter 6.

4.2 NURBS Toolbox for Matlab

The NURBS toolbox is a collection of Matlab functions which are useful in creating and manipulating NURBS curves and surfaces. The toolbox contains both Matlab script files and C routines. This helps in increasing the performance of the toolbox. All the NURBS routines are prefixed with “nrb” to differentiate them from inbuilt Matlab functions.

4.3 NURBS Skull for Matlab

The purchased NURBS skull model is in IGES format. IGES files cannot be directly opened in Matlab; therefore the data from the IGES file was extracted (i.e. the control points and the knot vectors) and stored in a Matlab compatible format. A data structure called “NurbsSkull” was created in Matlab as follows.

\[
\text{NurbsSkull (i)} \rightarrow \text{SurfaceName} \quad : \quad \text{A name given to the surface}
\]

\[
\text{NurbsSkull (i)} \rightarrow \text{ControlPoints (4, m, n)} \quad : \quad \text{3D Control points matrix}
\]

\[
\text{NurbsSkull (i)} \rightarrow \text{UKnots (1, m+3+1)} \quad : \quad \text{Knot vector in U direction}
\]

\[
\text{NurbsSkull (i)} \rightarrow \text{VKNots (1, n+3+1)} \quad : \quad \text{Knot vector in V direction}
\]

Where:

\( i \) is the number surface model. It varies from 1 through 46;

\( m \) is number of control points in U direction;

\( n \) is number of control points in V direction.
Each element of data structure contains all the information about one surface model. For example:

\[
\text{NurbsSkull (1) } \rightarrow \text{SurfaceName= “Mandible”}
\]

\[
\text{NurbsSkull (1) } \rightarrow \text{ControlPoints (4, 24, 61) = A 3D Control points matrix of size 4 X24 X 61}
\]

\[
\text{NurbsSkull (1) } \rightarrow \text{UKnots (1, 28) : Knot vector in U direction}
\]

\[
\text{NurbsSkull (1) } \rightarrow \text{VKNots (1, 65) : Knot vector in V direction}
\]

### 4.4 Interface

The interface was built within Matlab [Matlab®], which has a powerful graphical user interface development environment called GUIDE. The developed interface has various tools such as virtual camera controls and lighting controls which facilitate better visualization of the 3D models. It is also equipped with plot editing tools which are helpful in modifying various model properties such as color, transparency, type of projection used (perspective / orthographic). File format used for this research is .MAT. However other file formats can be incorporated easily.
Chapter 5. Automatic 3D Model to Scanned Data Registration

Registration is the process that aligns a 3D surface model to the data acquired from 3D scanning system. A good 3D model registration is very important to facilitate automation of forensic facial reconstruction process because this will reduce the number of iterations required in the fitting process and in turn help with automatic feature identification and placement of landmarks.

A number of attempts have been made and several algorithms exist for automatic registration. However, no robust method has been reported for aligning models to data. An alternative method for automatic registration of a 3D model to data is therefore introduced in this chapter. It does not require any partial overlap or preregistration [Chetverikov, 2002], does not require any user interaction and works well for both symmetric as well as non-symmetric models. The basic concept and assumptions of the automatic alignment algorithm and its application to the skull body and mandible are discussed in this chapter.

5.1 Concept

One basic premise of this approach is that a canonical frame of reference can be developed which characterizes the shape of the model. In addition, two 3D objects having similar shape should have a similar canonical frame of reference even when the model and dataset have different sizes but essentially similar shapes.

Based on this concept a canonical or natural frame of reference, with the centroid as the origin, can be defined for both the skull model and datasets.
When the canonical axes of the model and data are brought together, characteristic features of both should also be aligned, for example the point of maximum distance from the centroid will be at approximately the same relative location in both the model as well as the dataset. The key to this problem then is finding the proper local frame of reference. Once defined, registration of the model to the data simply becomes a transformation of its coordinates to a new frame of reference.

5.2 Basic Assumptions

The only assumption made for this algorithm is that the shape of the model and data should be similar. This implies that the scanned data should be a reasonably complete model and should not have any outliers.

5.3 Canonical Reference Frame

The first problem to be addressed is defining a logical, orthonormal coordinate system for the 3D model. The centroid is a logical choice of origin since the centroids of the model and the data will approximately lie at the same respective location. Defining three orthonormal axes starting at the origin is the more challenging task.

In case of symmetric models like the mandible, the normal to the plane of symmetry is an intuitive choice for one axis. The other two axes must be perpendicular to this normal and hence must lie in the plane of symmetry.

Due to symmetry the points of maximum and minimum distance from the centroid, shown as stars respectively in Figure 5.1, come in pairs and lie at the same distance from the centroid but on opposite side of the symmetry plane.
This sets up a clear mechanism for determining the front versus back of the mandible using the distance from the centroid. The midpoints of line between these extreme points, as depicted in Figure 5.1, roughly lie on the plane of symmetry and are used in formation of the canonical frame of reference as explained in next section.

![Figure 5.1 Canonical frame of reference for NURBS mandible](image)

**5.4 The Algorithm for Formation of Canonical Reference Frame**

The steps involved in formulation of a canonical coordinate axes are itemized below.

1. Calculate the centroid of the dataset.
2. Identify the furthest and closest point from the centroid on either side of the symmetric plane.

3. Compute the pseudo-normal to the symmetry plane

4. Choose the proper symmetry plane normal using the pseudo-normal.

5. Formulate the coordinate axes.

This algorithm is applied only once for the NURBS model at which stage then control point data is transformed into its canonical coordinate system. The same basic algorithm is then applied to each subsequent scanned dataset. The 3D model and scanned data are effectively automatically registered once transformed to their local coordinate frame.

The steps involved in mandible reference frame formation are discussed in the sections to follow. Figure 5.2 illustrates the initial position and orientation of NURBS model and the scanned data in world coordinate system. This set up is used to demonstrate each step in the algorithm.

Figure 5.2 A NURBS model (gray) and mandible data model (copper) in world coordinate axes with X (red), Y (blue) and Z (green)
5.4.1 Centroid Calculation

The centroid, as shown in Figure 5.3, is chosen as the natural origin for the canonical coordinate system. A good data set, having no missing data and outliers is required for best results. A uniform point distribution dataset is created in order to improve the centroid calculation by using an inbuilt Matlab function reducepatch. For this application, the centroid, \( c \), is calculated by simply averaging all the point coordinates in the dataset, \( D \) using the equation 5.2,

\[
D = \{ v_i \}, \quad i=1…n
\]  
\[
c = \frac{1}{n} \sum_{i=1}^{n} v_i
\]

where:

- \( D \) is the data set;
- \( v_i \) are the vertices \([X,Y \text{ and } Z]\);
- \( c \) is a point vector containing coordinates of the centroid;
- \( n \) is number of points in the data set.

(a)                                                                (b)

Figure 5.3 (a) Centroid of NURBS model (b) Centroid of data model
5.4.2 Identification of Furthest and Closest Points from Centroid on Either Side of Symmetric Plane

Since the mandible is symmetric it is essential to isolate the data in order to characterize the shape of the model. The plane of symmetry is not known; however, since the model is symmetric it is reasonable to assume that the centroid lies on the plane of symmetry.

The distances of each data point from the centroid are calculated, sorted in descending order of the distance and are stored in an array \( S \). The distances \( S_{\text{MAX}} \) and \( S_{\text{MIN}} \) correspond to the points of maximum and minimum distances from the centroid. \( P \) is list of data points sorted in descending order of \( S \). The first 5% of the list includes maximum distance points on both the side of the symmetric plane and similarly, the last 5% of the list includes minimum distance points on both the sides of symmetry plane. 5% was chosen on experimental bases.

Now, to separate extreme points across the plane of symmetry a second set of difference (\( L_{\text{MAX}} \) and \( L_{\text{MIN}} \)) is calculated relative to the points \( P_{\text{MAX}} \) and \( P_{\text{MIN}} \) respectively as shown in equation 5.4.

\[
S = \{S_{\text{MAX}}, S_2, S_3 \ldots S_m \ldots S_{n-m+1} \ldots S_{n+m+2}, S_{n-m+(m-1)}, S_{\text{MIN}}\} \quad (5.3)
\]

\[
L_{\text{max}} = \{ || p_i - P_i || \} \quad i = 1 \ldots m \\
L_{\text{min}} = \{ || P_{n-m+i} - P_i || \} \quad i = 1 \ldots m \\
\]

\( (m \text{ is 5% of } n) \)

The largest successive gap in difference between points indicates the separation across symmetry plane as shown in equation 5.5.
\[ I_1 = \text{maximum} \left( \Delta \left( L_{\text{max}(p)} - L_{\text{max}(p-1)} \right) \right) \]
\[ I_2 = \text{maximum} \left( \Delta \left( L_{\text{min}(q)} - L_{\text{min}(q-1)} \right) \right) \] (5.5)

At this stage the points which lie on mandibular condyle are determined. They are at maximum distance from the centroid, \( P_{\text{max}1} \) and \( P_{\text{max}2} \), and on either side of symmetry plane and similarly points of minimum distance from the centroid, \( P_{\text{min}1} \) and \( P_{\text{min}2} \), on either side of symmetry plane are determined which lie on mylohyoid line. In Figure 5.4, points at maximum distance from centroid are shown by stars and points at minimum distance are shown as diamonds.

![Figure 5.4 Points having maximum and minimum distance from centroid](image)

(a) NURBS model   (b) Scanned data

5.4.3 Computation of Pseudo-Normal to the Symmetry Plane

It can be seen from Figure 5.5 that the midpoint of \( P_{\text{max}1} \) and \( P_{\text{max}2} \), say \( P_{\text{max}} \) and the midpoint of \( P_{\text{min}1} \) and \( P_{\text{min}2} \), say \( P_{\text{min}} \) lie on the symmetry plane and are represented by squares and triangles respectively as shown in Figure 5.5.
Since $\mathbf{P}_{\text{max}}$ and $\mathbf{P}_{\text{min}}$ are the mid points, like the centroid, theoretically they should lie on the plane of symmetry; however, these points do not precisely lie on the symmetry plane and hence the normal computed using these points is only a pseudo-normal which is used to identify the positive side of symmetry plane.

$\mathbf{v}_{\text{max}}$ and $\mathbf{v}_{\text{min}}$ can now be constructed from these three points see Figure 5.6. $\mathbf{v}_{\text{max}}$ starts from the centroid and ends at $\mathbf{P}_{\text{max}}$ and $\mathbf{v}_{\text{min}}$ from centroid to $\mathbf{P}_{\text{min}}$.

\[
\begin{align*}
\mathbf{v}_{\text{max}} &= \mathbf{P}_{\text{max}} - \mathbf{c}, \\
\mathbf{v}_{\text{min}} &= \mathbf{P}_{\text{min}} - \mathbf{c}
\end{align*}
\]
Figure 5.6 Vectors $V_{\text{max}}$ and $V_{\text{min}}$. (a) NURBS model b) Scanned data

Vectors $v_{\text{max}}$ and $v_{\text{min}}$ lie on and thus define the pseudo-symmetry plane. Hence a pseudo-normal to this plane, $n$ is essentially a cross product of $v_{\text{max}}$ and $v_{\text{min}}$ and is represented by a green vector in Figure 5.7

$$n = v_{\text{max}} \times v_{\text{min}}$$  \hspace{1cm} (5.8)

Figure 5.7 Normal to the plane of symmetry (a) NURBS model (b) Scanned data
As it can be seen in Figure 5.7(b) the pseudo-normal in the scanned data is offset from the actual normal position and hence it is required to compute the real normal in order to get best results.

![Figure 5.8 Normal vectors n1 and n2 in black and green color](image)

**5.4.4 Computation of Appropriate Normal**

A normal to a plane is any vector perpendicular to that plane. Two vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ as shown in Figure 5.8 can be constructed from $\mathbf{P}_{\text{max}}$ as follows:

$$
\mathbf{n_1} = \mathbf{P}_{\text{max}} - \mathbf{P}_{\text{max}}
$$

$$
\mathbf{n_2} = \mathbf{P}_{\text{max}} - \mathbf{P}_{\text{max}}
$$

Points $\mathbf{P}_{\text{max}}$ and $\mathbf{P}_{\text{max}}$ are on opposite side of symmetry plane and also equidistant from the centroid hence the midpoint of line $(\mathbf{P}_{\text{max}}$, $\mathbf{P}_{\text{max}})$, $\mathbf{P}_{\text{max}}$ is essentially at the shortest distance from centroid. This implies that vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ are perpendicular to the actual symmetry plane.

Pseudo-normal $\mathbf{n}$ gives the positive side of the symmetry plane. Its dot product either with $\mathbf{n_1}$ or $\mathbf{n_2}$ must be positive depending upon which ones lie on
the positive half of the symmetry plane. The normal vector corresponding to the positive dot product is chosen as a normal to the plane of symmetry as shown in Figure 5.9.

\[ S_1 = n_1 \cdot n \] 
If \( S_1 \) is positive \( n_1 \) is the new normal \( n_{\text{new}} \).

\[ S_2 = n_2 \cdot n \] 
If \( S_2 \) is positive \( n_2 \) is the new normal \( n_{\text{new}} \). \hfill (5.10)

Figure 5.9 New corrected normal is shown with green color (a) NURBS model (b) Scanned data

5.4.5 Formation of Canonical Reference Frame

A coordinate system can be defined by an origin and three orthonormal vectors starting at the origin. The centroid of the model is used as the origin and two mutually perpendicular vectors \( v_{\text{max}} \) and \( n_{\text{new}} \) are chosen as two of the necessary axes. The third orthonormal vector \( t \) can be easily obtained by the cross product of \( v_{\text{max}} \) and \( n_{\text{new}} \). In Figure 5.10 vectors \( v_{\text{max}} \), \( n_{\text{new}} \) and \( t \) shown in blue, red and green color respectively

\[ t = n_{\text{new}} \times v_{\text{max}} \] \hfill (5.11)
Local coordinate system for NURBS model and scanned data are formulated. The origin of the new coordinate system is at centroid, and the three orthonormal axes X, Y, and Z are represented by vectors $n_{\text{new}}, v_{\text{max}}$, and $t$ respectively.

Figure 5.10 Formulation of coordinate axes: (a) NURBS model (b) Scanned data

Figure 5.11 shows a NURBS model and scanned data along with their respective canonical coordinate systems in the world coordinate system with X-axis in red, Y-axis in blue and Z-axis in green color.

Once a coordinate system is formulated it only takes a Euclidian transformation to align the scanned data to the 3D model a $4 \times 4$ coordinate transformation matrix (CTM) is constructed as follows:

$$\begin{bmatrix}
    a & e & i & m \\
    b & f & j & n \\
    c & g & k & o \\
    d & h & l & p
\end{bmatrix} \quad (5.12)$$

where:
a, b and c are the X, Y and Z components of the normal \( \mathbf{n} \);
e, f and g are the X, Y and Z components of the vector \( \mathbf{v}_{\text{max}} \);
i, j and k are the X, Y and Z components of the vector \( \mathbf{t} \);
d, h, l are dot products of centroid \( \mathbf{c} \) with normal \( \mathbf{n} \), vector \( \mathbf{v}_{\text{max}} \) and vector \( \mathbf{t} \) respectively;
m, n and o are zeros and p is one.

Figure 5.11 NURBS model, scanned data, their respective canonical coordinate systems in the world coordinate system

The transformation of NURBS model and scanned data from world coordinate system to their respective canonical coordinate axes is shown in Figure 5.12 and 5.13 respectively.
The NURBS model is automatically aligned with the scanned data when both are transformed to their respective canonical frame of reference from world coordinate system as shown in Figure 5.14.
5.5 Automatic Alignment of the Skull Body

The algorithm is applied on NURBS skull body. Figure 5.15 and 5.16 shows the position and orientation of a NURBS skull model and a scanned dataset in world coordinates. The X, Y and Z axis in world coordinate system are represented by bold red, blue and green color arrows respectively. The narrow arrows are used to depict canonical coordinate systems.
The skull body is a relatively complex 3D object to scan which results in uneven scanned data. This may bias the centroid calculation which in turn causes error in coordinate axes formulation. In spite of these errors, the algorithm shows reasonable results, which are enhance through closest point search methods along with the volume based scaling as discussed in the next chapter. Figure 5.17 shows the alignment of the scanned skull to the NUBRS skull.

Figure 5.16 Scanned skull model with world and local coordinate system

Global and localized deformation can now be applied on the NURBS mandible and skull models in order to better fit the model to the scanned data. These techniques are discussed in Chapter 6.
Figure 5.17 NURBS model and scanned data transformed to world coordinate system
Chapter 6. Fitting Based on Deformation of Pseudo-Isoparametric Curves for 3D Data Fitting

Once a model is aligned to the dataset, as discussed in the previous chapter, further refinement is required to adjust the model to fit the size and shape features of the data. A few existing deformation methods for fitting a model to a point cloud are discussed in Chapter 2. An alternative method for fitting a NURBS model to 3D point cloud is discussed in this chapter.

As noted in Chapter 2, the mandible bone is a single model whereas the remainder of the skull body is a collection of surface models which demand different fitting techniques for each. The global scaling and deformation techniques and the methodology of local deformation of mandible and skull are discussed in this chapter.

6.1 Basic Concept

Registration brings the model into alignment with the data. However, the size and shape of the model and data generally differ up to a certain degree where features do not precisely match. The mandible and skull body models are first scaled and deformed globally to improve the initial fit and then the models are deformed locally. The feature-based fitting technique developed by Gregory Dobson [DOBSON, 1997] takes advantage of the known geometric shapes. Based on similar concept, an alternative fitting technique is developed.

6.2 Fitting the Mandible

The basic steps involved in fitting the mandible are global scaling and global deformation and then local fitting of model by deformation of pseudo-isoparametric curves.
6.2.1 Global Scaling of the Mandible

The NURBS model and the data are aligned using euclidian transformations as discussed in section 5.5 but the size and shape of the model differ from that of the data. Hence the NURBS model is scaled and deformed to improve alignment.

Independent scaling in the X Y and Z directions based on a max/min box calculation cannot be used directly because the alignment of axes do not necessary align the corresponding features. A uniform scaling factor is required. A scale factor based on the averaging ratios of volume and the ratios of maximum and minimum distances is taken. Equation 6.1

\[
S = \left( \frac{\frac{1}{3} \left( \frac{1}{V_{\text{Model}}} + \frac{(V_{\text{Max}})_{\text{Data}}}{(V_{\text{Max}})_{\text{Model}}} + \frac{(V_{\text{Min}})_{\text{Data}}}{(V_{\text{Min}})_{\text{Model}}} \right)}{3} \right)
\]  

(6.1)

In this equation:

- \(V_{\text{Data}}\) is data volume;
- \(V_{\text{Model}}\) is NURBS model volume;
- \((V_{\text{Max}})_{\text{Data}}\) is maximum distance vector from centroid for the data;
- \((V_{\text{Max}})_{\text{Model}}\) is maximum distance vector from centroid for the model;
- \((V_{\text{Min}})_{\text{Data}}\) is minimum distance vector from centroid for the data;
- \((V_{\text{Min}})_{\text{Model}}\) is minimum distance vector from centroid for the model.

The volume ratio incorporates 3D global scaling hence cube root of the ratio is used to obtain a average scale factor for each dimension. The volume of the model and the data is computed using [Lein, 1984] for faceted model.
6.2.2 Global Deformation of the Mandible

A global deformation is now applied to improve the overall model alignment. Using Delaunay’s closest point search technique [Barber, 1996], data-points closest to the control points of the mandible are identified and used to formulate the mapping. The transformation matrix for global deformation is calculated as follows:

\[ [X] = M^*D \]  \hspace{1cm} (6.2)

where:

- \( M \) is the \( N \times 4 \) matrix of control point of NURBS model;
- \( D \) is the \( N \times 4 \) matrix of scanned data point;
- \( N \) is number of control points in NURBS model;
- \([X]\) is the \( 4 \times 4 \) affine transformation used to deform the NURBS model.

\( M \) is a \( N \times 4 \) matrix and hence not invertible. As discussed in section 2.1, a pseudo-inverse of \( M \) is calculated using singular value decomposition. As shown in Figure 6.1, the alignment of NURBS model to the scanned data is improved by the global scaling and deformation. The NURBS model is now ready for local tweaking to better fit the scanned mandible.

6.2.3 Isoparametric Curve of the Mandible

The model is now to be deformed locally using the isoparametric curves depicted in Figure 6.2. The control mesh of NURBS is treated as isoparametric curves, and hence change in shape of these curves directly changes the shape of the model. Figure 6.2 depicts the isoparametric curves of mandible.
Figure 6.1 (a) Before and (b) After global scaling and deformation
6.2.4 Translation of Isoparametric Curve of the Mandible

The isoparametric curves are translated individually in order bring curves closer to the dataset. Translation of curves does not change the shape of the curve thus maintaining the inherent features of the model. Translation of curves also helps in better point association when local curves are deformed locally.

A delaunay’s closest point search technique [Barber, 1996] is used for obtaining the closest data point associated to the control points on isoparametric curve. This closest point search is a computationally expensive process and hence to increase efficiency only those data points enclosed in a scaled max-min box of the isoparametric curve are used.
Figure 6.3 (a) Error vectors (b) Average error
An isoparametric model contour $M$, consisting of $n$ control points is depicted as red curve and the blue stars represent the dataset $D$ with $n$ points. A translation is calculated by averaging the error vectors associated with all the points in the curve. Figure 6.3 shows the error vectors and the average error vector used for constructing translation matrix. The NURBS points on the isoparametric curves are then moved using this average error vector as the translation.

This process is iteratively applied until the translation vector is zero or negligible. As seen in Figure 6.3 (b) the average error in translation is very small and diminishes with increasing iterations. By experience, three iterations is typically sufficient. In Figure 6.4, the resulting model fit after applying the local translation to the isoparametric contours is illustrated.

It can be seen from Figure 6.4 that the model fit is improved by contour translation and the model is moved closer to the data. Local tweaking is now applied to the isoparametric curves.

**6.2.5 Deformation of Isoparametric Curves of the Mandible**

The point associations are obtained using the Delaunay’s closest points search method. Only the data-points which fall within the scaled max-min box are used for obtaining the association. The point associations at this stage are more accurate since translation brings the model closer to the data-points. The transformation matrix for the deformation of isoparametric curve is obtained using equation 6.3.
Figure 6.4 Model fitting after applying translation to isoparametric curves

\[ \begin{bmatrix} X \end{bmatrix} = M_i^* D_i \]  \hspace{1cm} (6.3)

where:

- \( M_i \) is the \( n \times 4 \) matrix of points on isoparametric curve;
- \( D_i \) is the \( n \times 4 \) matrix of closest points to the isoparametric curve;
- \( n \) is number of control in NURBS model;

Figure 6.5 demonstrate the fitting of model to data set by deformation of isoparametric curves. The features of the skull can now be identified. Feature detection is discussed along with results in Chapter 7.
6.3 Fitting Process for the Skull

The fitting of the skull body proceeds with similar steps as that of mandible with a few exceptions. Isoparametric curves cannot be used directly for the translation and deformation. The skull is first globally scaled and deformed using the techniques discussed in section 6.2. Instead of translating isoparametric curves, surface models are grouped together and translated in order to align features. Isoparametric curves, like contours, are then developed as explained later in this section. These contours are used for local deformation purpose.

6.3.1 Global Scaling of the Skull

The model skull and the data skull are already registered in the same manner as the mandible. The next step in the fitting process is to scale the model
skull and deform it globally to facilitate local fitting. The scaling factor for skull is calculated in the same fashion as used for the mandible.

The scaling factor is calculated using equation 6.1. Figure 6.6 (a) and (b) demonstrates a skull model before and after scaling respectively.

6.3.2 Global Deformation of the Skull

The global alignment method discussed in section 6.2.2 is applied to the skull model. Control points of all the surface models are appended to one single matrix, Delaunay’s closest point search algorithm is then used to obtain the closest data points to associate with these control points. The transformation for deforming the model globally is then calculated using SVD and equation 6.3. Figure 6.7 depicts the skull body model which is deformed globally.

Figure 6.6 (a) Skull model before scaling
Figure 6.6 (b) Skull model after scaling

Figure 6.7 Globally deformed skull model
1. As shown in Figure 6.8 (a) Frontal bone is not grouped with other models since the model is larger and smoother compared to the others.

2. The left and right eye orbits and nasal bone are grouped as shown in Figure 6.8 (b).

3. The left and right malar and occipital are grouped together to form second set as illustrated in Figure 6.8(c).

4. The left and right zygomatic bone, left and right maxilla, anterior nasal spine and nasal cavity from another group as depicted in Figure 6.8 (d).

5. As demonstrated in Figure 6.8(e) temporal bone is not grouped with other models since the model is larger in size compared to other models and also it has uncomplicated shape.

### 6.3.3 Translation of Grouped Models

The model is now within a tolerance level of the data such that acceptable point association can be obtained from closest point search. The next step is to bring features in closer tolerance. This is done by transforming the surface models in groups.

Translating all the surface models individually may result in random translation. Hence symmetric models are grouped before translation. The grouping of the models is shown in Figure 6.8.

Each group is considered individually for translation. Using Delaunay’s closest point search algorithm, closest points for all the points the group are obtained. An error is calculated for each association and the group is translated using the average of these error vectors. Figure 6.9 demonstrates the results.
after translating these groups individually. As it can be seen in Figure 6.9 the features moved closer to the model data. However the continuity of the model is lost.

Figure 6.8 Grouping of models for translation. (a) Frontal bone (b) Left and right eye orbit and nasal bone (c) Left and right malar and occipital (d) The left and right zygomatic bone, left and right maxilla, anterior nasal spine and nasal cavity (e) temporal bone
6.3.4 Formulation of Pseudo-Isoparametric Curves

As mentioned earlier, the skull model is a complex geometry and hence is created as a collection of surface models with geometric continuity at the boundaries. Since each surface model has its own parametric domain, deforming the isoparametric curves of the model may lead to surface model discontinuities. Pseudo-isoparametric curves are thus created for deformation purposes.

The skull is placed in a horizontal orientation as shown in Figure 6.10. Pseudo-isoparametric curves or contours have approximately constant value of Y-coordinate. The contours for skull model are shown in Figure 6.10.
6.3.5 Deformation of Contours of Skull Model

The deformation process for skull is similar to the deformation process of mandible except the horizontal contours are deformed instead of principal isoparametric curves. Figure 6.11 demonstrates the model after deformation of contours.

Figure 6.10 Contours of NURBS skull

Figure 6.11 Final fitting of skull model by local deformation of pseudo-isoparametric curves
Chapter 7. Algorithm Testing and Results

In previous chapters, algorithms for automatic registration of a 3D model and model fitting by local deformation are discussed. These algorithms were tested for reproducibility and repeatability and the results are presented in this chapter. A skull body and mandible NURBS models are aligned and fit to several scans of the same physical artifacts to check for reproducibility.

The objective of registration is to bring model is close proximity with the dataset which will facilitate model fitting at a later stage. Amongst the existing automatic registration methods, principal axes alignment is most widely accepted except that it may produce ambiguous results in case of symmetric models [Dobson, 1997]. The automatic registration algorithm developed in this thesis is verified by comparing the axes developed to the model and data \textit{principal axes}. The registration algorithm was tested for alignment of the mandible and the skull. These results are in section 7.1 and 7.2 respectively.

As mentioned earlier, the of mandible and skull model structure are different and hence the fitting algorithm was customized. The results achieved with these processes are discussed in section 7.3 and 7.4 respectively.

Maximum and average error in surface fit are the computed distances of each point in the dataset to its closest point on the NURBS model. One of the goals of this thesis is to identify features/landmarks from scanned data which are required for facial reconstruction.
After model fitting, data landmarks are obtained by finding the closest data points to the landmarks on the NURBS model. These are then compared to interactively selected landmarks on the dataset.

7.1 Registration of Mandible

For repeatability and reproducibility, nine scanned mandible datasets with varying numbers of 3D points were used. These datasets were obtained from three different mandibles. Table 7.1 briefly summarizes these datasets.

<table>
<thead>
<tr>
<th>JAW</th>
<th>Number of control points</th>
<th>Number of Facets</th>
<th>Model Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaw 1-1</td>
<td>159261</td>
<td>283116</td>
<td>Raw data with no outliers</td>
</tr>
<tr>
<td>Jaw 1-2</td>
<td>5303</td>
<td>10223</td>
<td>Processed data with no outliers</td>
</tr>
<tr>
<td>Jaw 1-3</td>
<td>22840</td>
<td>45495</td>
<td>Processed data with no outliers</td>
</tr>
<tr>
<td>Jaw 2-1</td>
<td>231858</td>
<td>377269</td>
<td>Raw data with outliers</td>
</tr>
<tr>
<td>Jaw 2-2</td>
<td>62385</td>
<td>124198</td>
<td>Processed data with no outliers</td>
</tr>
<tr>
<td>Jaw 2-3</td>
<td>53802</td>
<td>106055</td>
<td>Processed data with no outliers</td>
</tr>
<tr>
<td>Jaw 3-1</td>
<td>207263</td>
<td>373569</td>
<td>Raw data with outliers</td>
</tr>
<tr>
<td>Jaw 3-2</td>
<td>356031</td>
<td>712058</td>
<td>Processed data with no outliers</td>
</tr>
<tr>
<td>Jaw 3-3</td>
<td>5731</td>
<td>11458</td>
<td>Processed data with no outliers</td>
</tr>
</tbody>
</table>

7.1.1 Registration Results

Registration results for the nine mandibles are presented in Figures 7.1 (a) through 7.9 (b). Datasets with outliers, namely Jaw 2-1 and Jaw 3-1, are also tested. These datasets are not appropriately registered since they violate the basic assumption which is “the model and dataset should have similar geometric shape but” but are included to illustrate their impact. Figures 7.4 and 7.7 depict the results of registration of datasets with outliers.
The registration algorithm aligns the canonical axes of the model and the dataset. The principal axes of the model and data are then computed for comparison. The angle between the corresponding axes is compared for orientation errors. The following color code is used for distinguishing different axis.

The red, blue and green colors represent the X, Y and Z canonical axes of the model and the data. The dotted black, brown and the gray represent the principal X, Y and Z axes of the NURBS model where as the dotted yellow cyan and magenta represent the principal X, Y and Z axes of the dataset.

7.1.2 Comparison of the Canonical with Principal Axes

The model is aligned to the dataset using their canonical axes. The principal axes of the model and data are also computed and are depicted in Figure 7.1 through 7.9.
Figure 7.1  (a) Jaw 1-1 before registration (b) Jaw 1-1 after registration
Figure 7.2 (a) Jaw 1-2 before registration (b) Jaw 1-2 after registration
Figure 7.3 (a) Jaw1-3 before registration
Figure 7.3 (b) Jaw 1-3 after registration
Figure 7.4 (a) Jaw 2-1 before registration. Outliers circled
Figure 7.4 (b) Jaw 2-1 after registration
Figure 7.5 (a) Jaw 2-2 before registration
Figure 7.5 (b) Jaw 2.2 after registration
Figure 7.6 (a) Jaw 2-3 before registration

Figure 7.6 (b) Jaw 2-3 after registration
Figure 7.7 (a) Jaw 3-1 before registration. Outliers circled

Figure 7.7 (b) Jaw 3-1 after registration. Outliers circled
Figure 7.8 (a) Jaw 3-2 before registration
Figure 7.8 (b) Jaw 3-2 after registration
Figure 7.9 (a) Jaw 3-3 before registration
The comparison is done of the angles between the individual canonical axes and the corresponding principal axes of the model and the data. It should be noted that the principal axes are depicted in their computed orientation which may be the ambiguity noted earlier but are mirrored for error comparison.

The orientation of the canonical axes is compared with that of the principal axes. First, as shown in Table 7.2, the angles between the respective principal axes of the model and the data are compared, then the angles between the principal axes and the canonical axes of model and data are compared as presented in Table 7.3.
The model and data are registered by aligning the canonical axes and hence the angle between the respective canonical axes of the model and the data is zero. However, the angle between the respective principal axes ranges from 2 to 20 degrees except for Jaw 2-1 and Jaw 3-1 which are not registered as expected due to outliers. In case of Jaw 2-2 the error in angles is large; however, as shown in Figures 7.5 (a) and (b) the model is suitably registered to the data. Higher values of angle indicate that the points in dataset are unevenly distributed which plays important role in principal axes calculation and hence is one of the causes of error.

The angle between the canonical axes and the principal axes of NURBS model is less than 2 degrees as shown in Table 7.3. The control point distribution in the NURBS model is uniform and hence this shows that the canonical axes and the principal axes of any 3D object are in proximity. The angles between the canonical axes and principal axes of dataset range from 2 to 20 except for Jaw 2-1 and Jaw 3-1 since these datasets are not registered as expected due to outliers. The large angle values are due to uneven point distribution.

It can be seen from Figure 7.1 through 7.9, all the models except Jaw 2-1 and Jaw 3-1(dataset with outliers) are suitably registered with the NURBS model and the principal axes is aligned to the canonical axes in acceptable tolerance. The goal or automatic registration algorithm is to roughly align the model and dataset to facilitate the process of final fitting.
Table 7.2 Comparison of angle between principal axes and canonical axes of NURBS jaw and scanned mandible

<table>
<thead>
<tr>
<th>Jaw</th>
<th>Angle (in degrees) Between Principal Axes of Model and Data after Registration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>About X axis</td>
</tr>
<tr>
<td>Jaw 1-1</td>
<td>2.1</td>
</tr>
<tr>
<td>Jaw 1-2</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 1-3</td>
<td>9.9</td>
</tr>
<tr>
<td>Jaw 2-1 (With Outlier)</td>
<td>68.2</td>
</tr>
<tr>
<td>Jaw 2-2</td>
<td>17.0</td>
</tr>
<tr>
<td>Jaw 2-3</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 3-1 (With Outlier)</td>
<td>37.7</td>
</tr>
<tr>
<td>Jaw 3-2</td>
<td>15.1</td>
</tr>
<tr>
<td>Jaw 3-3</td>
<td>8.4</td>
</tr>
</tbody>
</table>
Table 7.3 Angles between the principal axes and canonical axes of NURBS jaw and scanned mandible (in degrees)

<table>
<thead>
<tr>
<th>Jaw</th>
<th>Angle (in degrees) between the Principal and Canonical Axes of Model</th>
<th>Angle (in degrees) between the Principal and Canonical Axes of Data</th>
<th>Difference in angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>About X axis</td>
<td>About Y axis</td>
<td>About Z axis</td>
</tr>
<tr>
<td>Jaw 1-1</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 1-2</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 1-3</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 2-1 (With Outlier)</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 2-2</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 2-3</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 3-1 (With Outlier)</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 3-2</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Jaw 3-3</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
</tr>
</tbody>
</table>
7.2 Registration of the Skull Body

The registration process of the skull follows the same protocol as used for the mandible. Registrations of three skull models varying in shape and number of points are processed for repeatability and reproducibility. These datasets, obtained from scanning 3 different plastic skull models, are briefly described in Table 7.4. Figures 7.10 (a) through 7.12 (b) illustrate registration of skull followed by Tables 7.5 through 7.7 which describe the testing results.

Table 7.4 Scanned skull description

<table>
<thead>
<tr>
<th>Skull</th>
<th>Number of control points</th>
<th>Number of Facets</th>
<th>Model Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull 1</td>
<td>246094</td>
<td>489731</td>
<td>Raw data with no outliers</td>
</tr>
<tr>
<td>Skull 2</td>
<td>23172</td>
<td>46364</td>
<td>Processed data with no outliers</td>
</tr>
<tr>
<td>Skull 3</td>
<td>30471</td>
<td>60164</td>
<td>Processed data with no outliers</td>
</tr>
</tbody>
</table>

It can be observed from Figure 7.10 through 7.13, all the models usually appear suitably registered with the NURBS model and the principal axes are aligned to the canonical axes within acceptable tolerance.

The model and data are registered by aligning the canonical axes and hence the angle between the respective canonical axes of the model and the data are negligible; however, the angle between the respective principal axes ranges from 0.04 to 0.4 degrees. The results are shown in Table 7.5.

The angles between the canonical axes and the principal axes of NURBS model is less than 2 degrees as shown in Table 7.6. The angles between the canonical axes and principal axes of dataset range from 0.8 to 1.6 degrees. The difference between these angles is a measure of proximity of canonical axes and the principal axes. The results are shown in Table 7.6.
Figure 7.10 (a) Skull 1 before registration
The automatic registration algorithm produces acceptable preliminary registration for the skull, hence it can be concluded that this alignment is suitable candidate for alignment of many complex 3D objects.

Table 7.5 Angle between principal axes NURBS skull and scanned skull

<table>
<thead>
<tr>
<th>Skull</th>
<th>About X axis</th>
<th>About Y axis</th>
<th>About Z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull 1</td>
<td>0.30</td>
<td>0.44</td>
<td>0.32</td>
</tr>
<tr>
<td>Skull 2</td>
<td>0.24</td>
<td>0.24</td>
<td>0.05</td>
</tr>
<tr>
<td>Skull 3</td>
<td>0.28</td>
<td>0.28</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Figure 7.11(a) Skull 2 before registration
Figure 7.11 (b) Skull 2 after registration
7.3 Local Fitting Result for Mandible

The fitting algorithm was tested for six mandible datasets. Table 7.7 and 7.8 present the maximum and the average error in surface fit and the landmark detection before and after fitting respectively. The surface fit error is the
calculated distance of each point in the dataset to its closest point on NURBS model.

Extraction of landmarks is the goal of this process. Eleven landmarks are selected on the NURBS mandible. Corresponding landmarks on the scanned data were interactively selected. After the fitting process was completed the points in the dataset closest to the NURBS landmarks are located for comparison. The error between the automatically detected landmarks and the interactively selected ones is a measure of algorithms performance although some error is introduced in picking the points interactively. A more accurate measure would require placing landmarks on the skull / mandible before scanning and then comparing these feature locations to the automatically detected landmarks.

The model and data are made semi transparent for better visualization in the figure. Also the NURBS landmarks, data landmarks and the automatically detected landmarks are marked with green triangle, blue pentagonal star, and red diamond respectively.

It can be seen from Table 7.7 and 7.8 that the average error in surface fitting range from 0.97 to 1.45 mm. The significant difference between the average error and the maximum error shows that the number of points having error of the scale of maximum error are very few. The maximum surface fitting errors occur at the mandibular condyle. The translation applied to control points in these regions are not lateral because of the geometry of the curves.
Table 7.6 Angles between the principal axes and canonical axes of NURBS skull and scanned skull (in degrees)

<table>
<thead>
<tr>
<th>Skull</th>
<th>About X axis</th>
<th>About Y axis</th>
<th>About Z axis</th>
<th>About X axis</th>
<th>About Y axis</th>
<th>About Z axis</th>
<th>About X axis</th>
<th>About Y axis</th>
<th>About Z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull 1</td>
<td>1.6</td>
<td>1.7</td>
<td>1.1</td>
<td>1.5</td>
<td>1.4</td>
<td>0.8</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Skull 2</td>
<td>1.6</td>
<td>1.7</td>
<td>1.1</td>
<td>1.5</td>
<td>1.5</td>
<td>1.1</td>
<td>-0.0</td>
<td>-0.2</td>
<td>-0.0</td>
</tr>
<tr>
<td>Skull 3</td>
<td>1.6</td>
<td>1.7</td>
<td>1.1</td>
<td>1.5</td>
<td>1.5</td>
<td>1.0</td>
<td>-0.0</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
</tbody>
</table>
Table 7.7 Error in jaw fitting and automatic landmarks detection before model fitting (in mm)

<table>
<thead>
<tr>
<th>Jaw</th>
<th>Average error in surface fit</th>
<th>Maximum error in surface fit</th>
<th>Average error in landmarks</th>
<th>Maximum error in landmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaw 1-1</td>
<td>3.2</td>
<td>11.5</td>
<td>12.1</td>
<td>20.4</td>
</tr>
<tr>
<td>Jaw 1-2</td>
<td>3.8</td>
<td>12.0</td>
<td>10.7</td>
<td>14.9</td>
</tr>
<tr>
<td>Jaw 1-3</td>
<td>3.0</td>
<td>10.8</td>
<td>13.7</td>
<td>17.5</td>
</tr>
<tr>
<td>Jaw 2-1</td>
<td>2.8</td>
<td>13.1</td>
<td>15.4</td>
<td>19.1</td>
</tr>
<tr>
<td>Jaw 2-2</td>
<td>4.1</td>
<td>12.2</td>
<td>11.6</td>
<td>18.1</td>
</tr>
<tr>
<td>Jaw 3-1</td>
<td>3.7</td>
<td>12.8</td>
<td>18.9</td>
<td>24.3</td>
</tr>
</tbody>
</table>

Table 7.8 Error in jaw fitting and automatic landmarks detection after model fitting (in mm)

<table>
<thead>
<tr>
<th>Jaw</th>
<th>Average error in surface fit</th>
<th>Maximum error in surface fit</th>
<th>Average error in landmarks</th>
<th>Maximum error in landmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaw 1-1</td>
<td>0.9</td>
<td>7.3</td>
<td>6.2</td>
<td>14.9</td>
</tr>
<tr>
<td>Jaw 1-2</td>
<td>1.4</td>
<td>5.6</td>
<td>4.7</td>
<td>6.4</td>
</tr>
<tr>
<td>Jaw 1-3</td>
<td>0.9</td>
<td>5.9</td>
<td>6.5</td>
<td>11.0</td>
</tr>
<tr>
<td>Jaw 2-1</td>
<td>1.0</td>
<td>7.6</td>
<td>7.5</td>
<td>13.9</td>
</tr>
<tr>
<td>Jaw 2-2</td>
<td>0.9</td>
<td>5.3</td>
<td>5.2</td>
<td>7.9</td>
</tr>
<tr>
<td>Jaw 3-1</td>
<td>1.3</td>
<td>6.5</td>
<td>8.1</td>
<td>17.7</td>
</tr>
</tbody>
</table>

The average error in the automatic landmark detection varies from 4 mm to 8 mm. The maximum error are higher compared to average error, however it
should be noted that the data landmarks and the NURBS landmarks are selected interactively which may also be a source of error.

Figure 7.13 Jaw 1-1 final fitting and automatic landmark detection

Figure 7.14 Jaw 1-2 final fitting and automatic landmark detection
Figure 7.15 Jaw 1-3 final fitting and automatic landmark detection

Figure 7.16 Jaw 2-1 final fitting and automatic landmark detection
Figure 7.17 Jaw 2-2 final fitting and automatic landmark detection

Figure 7.18 Jaw 3-1 final fitting and automatic landmark detection
7.4 Local Fitting Results for Skull

The fitting algorithm for skull is based on similar concept of local deformation; however, as discussed in the previous chapter, the fitting algorithm is customized for skull fitting in order to accommodate the different structure of skull model. The fitting algorithm is tested with two scanned dataset of skulls. The surface fit error is the calculated distance of each point on the dataset to the closest point on the NURBS model.

Eleven landmarks are selected on the NURBS skull. Corresponding landmarks on the scanned data are interactively selected. The closest points to the NURBS landmarks on the dataset are obtained. The error between the automatically detected points and the interactively selected points is computed. However, the portion of source of error may be in picking the points interactively. A more accurate measure would be to place landmarks on the skull / mandible before scanning and then comparing it to the automatically detected landmarks.

As it can be seen in Figures 7.19 and 7.20 the NURBS skull is closely fit to data. The NURBS landmarks, data landmarks and the automatically detected landmarks are marked with triangle, pentagon, and diamond respectively.

It can be seen from Table 7.9 and 7.10 that the average error in surface fitting is in a range of 2 to 2.5 mm. However, maximum error in surface fit is at the scale of 20 mm. This error is because of the lack of continuity in the NURBS skull model. Surface models, when translated lose the continuity across the boundaries which results in gap between surface models. The data points in these gaps have the maximum surface fit error.
Table 7.9 Error in skull fitting and automatic landmarks detection before model fitting (in mm)

<table>
<thead>
<tr>
<th>Skull</th>
<th>Average error in surface fit</th>
<th>Maximum error in surface fit</th>
<th>Average error in landmarks</th>
<th>Maximum error in landmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull 1</td>
<td>14.9</td>
<td>40.2</td>
<td>24.9</td>
<td>38.5</td>
</tr>
<tr>
<td>Skull 2</td>
<td>16.5</td>
<td>36.2</td>
<td>27.3</td>
<td>39.6</td>
</tr>
</tbody>
</table>

Table 7.10 Error in skull fitting and automatic landmarks detection after model fitting (in mm)

<table>
<thead>
<tr>
<th>Skull</th>
<th>Average error in surface fit</th>
<th>Maximum error in surface fit</th>
<th>Average error in landmarks</th>
<th>Maximum error in landmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull 1</td>
<td>2.1</td>
<td>17.0</td>
<td>10.6</td>
<td>26.8</td>
</tr>
<tr>
<td>Skull 2</td>
<td>2.1</td>
<td>21.2</td>
<td>5.3</td>
<td>16.3</td>
</tr>
</tbody>
</table>

The average error in the automatic landmark detection varies from 5 to 11 mm. The average error is in acceptable tolerance since the dowels used for manual reconstruction process have diameter of 4mm. The maximum error are higher compared to average error, however it should be noted that the data landmarks and the NURBS landmarks are selected interactively and may be a possible source of error.
Figure 7.19 Skull1 final fitting and automatic landmark detection
7.5 Artificial Data Test

Using the NURBS model a uniform point cloud distribution is generated for testing the algorithm. A random translation error in a range of -4 to 4 mm is associated to each point along the surface normal. The points from the artificial data set closest to the NURBS landmarks are used for computing error in landmark detection. The dataset is then arbitrarily rotated and translated in 3D space. Automatic registration, local fitting and landmarks detection algorithms are
Table 7.11 Registration testing for artificial mandible data created from NURBS model.

<table>
<thead>
<tr>
<th>Jaw</th>
<th>Angle (in degrees) between the Principal Axes and Canonical Axes of Model</th>
<th>Angle (in degrees) between the Principal Axes and Canonical Axes of Data</th>
<th>Error in angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>About X axis</td>
<td>About Y axis</td>
<td>About Z axis</td>
</tr>
<tr>
<td>NURBS</td>
<td>1.6</td>
<td>1.6</td>
<td>0.2</td>
</tr>
<tr>
<td>NURBS</td>
<td>1.6</td>
<td>1.6</td>
<td>0.2</td>
</tr>
<tr>
<td>NURBS</td>
<td>1.6</td>
<td>1.6</td>
<td>0.2</td>
</tr>
<tr>
<td>NURBS</td>
<td>1.6</td>
<td>1.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>
then applied to this dataset and the results are discussed in Table 7.11 and 7.12. The error between the automatically detected landmarks and the original landmarks is computed. The location of these points in model is same as in artificial data and hence the error introduced only by the algorithm can be computed.

Table 7.12 Fitting and landmarks detection errors for artificial dataset

<table>
<thead>
<tr>
<th>Jaw</th>
<th>Average error in surface fit</th>
<th>Maximum error in surface fit</th>
<th>Average error in landmarks</th>
<th>Maximum error in landmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>NURBS 1</td>
<td>0.8</td>
<td>2.6</td>
<td>2.4</td>
<td>3.9</td>
</tr>
<tr>
<td>NURBS 2</td>
<td>2.3</td>
<td>0.8</td>
<td>3.5</td>
<td>6.8</td>
</tr>
<tr>
<td>NURBS 3</td>
<td>0.8</td>
<td>2.2</td>
<td>2.6</td>
<td>3.8</td>
</tr>
<tr>
<td>NURBS 4</td>
<td>0.9</td>
<td>2.8</td>
<td>3.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>

It can be observed from Table 7.11 that the error in difference of angles is less than errors in the scanned datasets. In Table 7.12, fitting errors and landmark detection errors are listed. Since landmarks are not picked interactively, the algorithm is the primary source of error. It can be seen that the error introduced by the algorithm is less than 5 mm which is the diameter of the dowels used for facial reconstruction. Algorithm shows better results for artificial datasets because the error in landmark detection due to interactive selection of landmarks was eliminated.
Chapter 8. Conclusion and Future Work

Automation of forensic facial reconstruction process is a challenging task and several researchers are working towards automating the process. Various steps involved in the reconstruction process are by now automated; however, the key to complete automation of the process is landmark identification and this was the motivation for this research. By fitting a NURBS model of mandible and skull body with known geometric structure and landmarks to scanned data, the location of the landmarks in the data is established. The fitting is achieved in two steps, first the model is automatically registered and then deformed globally and locally to fit the data.

Various methods exist for registration and deformation as presented in Chapter 2. However each has limitations, hence alternative methods for registration and model fitting were developed and presented in this research. The advantages and limitations of the developed algorithms are summarized in this chapter followed by future work required to improve the results. Limitations of the 3D scanner and NURBS model are also summarized.

8.1 Advantages

An alternative method for automatic registration of a 3D model to data is presented. It does not require any partial overlap or preregistration [Chetverikov, 2002], does not require any user interaction and works well for symmetric models unlike principal axes alignment [Lein, 1984]. The algorithm was verified by applying it to 7 scans from 3 different mandibles and 3 skull datasets as detailed in the previous chapter. A reasonable alignment is achieved.
After registration, the model is deformed locally to fit the data. The deformation is carried out by tweaking the pseudo-isoparametric curves (control mesh) of the model. The mandible NURBS model generates a good fit except at the mandibular condyle where the point translations are not in lateral plane.

Since the skull body model is a collection of adjacent surface models, continuity at the boundaries is lost after deformation. Even so, the landmarks on the scanned data are identified with a reasonable level of accuracy, in case of both mandible and skull body, as discussed in previous chapter. A better surface model of skull will improve its accuracy.

8.2 Limitation and Future Work

The goal of this research was to take a step forward towards automation of the forensic facial reconstruction process by developing a mechanism which automatically identifies landmarks on the 3D scanned data of skull. There are limitations to the use of registration and fitting algorithm which are discussed in this section. The NURBS model used for the research also has some limitations are as summarized.

8.2.1 Automatic Registration

The canonical axes alignment method for automatic registration produces good results for symmetric as well as non symmetric models. However the data set, to which the model is to be aligned, needs to be free from outliers and should be a complete point set. Various methods for removing outliers exist and can be easily implemented. Scaioni suggested techniques based on least median squares for outlier rejection in automatic aerial triangulation [Scaioni, 2001].
The symmetric plane normal calculation becomes unstable when the centroid is too close to one of the surface points. Also if the centroid and the points of maximum and minimum distance are collinear the resulting cross product will be zero hence normal to the plane of symmetry cannot be calculated. This is however not an issue for mandible and skull models however may a problem for more general application.

The algorithm uses first and last 5% of the points from a list which is sorted based up on their distance from centroid. “5%” is chosen on experimental bases using 9 mandible datasets and 3 skull datasets. More validation is required for generalization of automatic registration algorithm.

8.2.2 Model Fitting by Deformation of Pseudo-Isoparametric Curve

Pseudo-isoparametric curves are contours of control points of the models. It is required that all the curves should have same parametric dimension in order to maintain continuity at the time of deformation. If a NURBS model is built using single surface patch, as was the mandible, the deformation of these curves results in a good fit; however, if the model is a collection of surface patches as is the case with the skull body, this technique cannot be used efficiently since the isoparametric curves of all the surface patches do not have same parametric dimension. A more robust model can be developed which is made of one NURBS surface patch and has same parametric dimension

8.2.3 3D Scanning System

The Polhemus FastSCAN cobra is an easy to use scanner however a certain level of skills is required by the person who is scanning since this requires
human interaction. The scanner is very sensitive and a smallest movement in the transformer leads to errors in point data collection. The software allows user to unselect unwanted scans from a list before further processing; however, no control is provided to remove a particular portion of data.
Bibliography


Vita

Hemant Narendra Khatod was born on September 26, 1979, in Medshi, India, to Narendra and Savita Khatod. He grew up in Aurangabad and attended Holy Cross English High School. In 1997, he graduated high school. He earned his Bachelor of Engineering Degree in Mechanical Engineering from Dr. B. A. Marathwada University in June 2001. Immediately after graduation, Hemant entered the graduate program at Louisiana State University. His areas of interest are computer aided design and finite element analysis. He successfully defended his thesis on November 5, 2004 and will receive the degree of Master of Science in Mechanical Engineering in December 2004.