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Note

On the Matroids Representable over $GF(4)$

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The purpose of this note is to present a counterexample to a conjecture of Kahn and Seymour on the minor-minimal matroids not representable over $GF(4)$.

Kahn and Seymour [2] have conjectured that a matroid is representable over $GF(4)$ if and only if it has no minor isomorphic to any of the matroids $U_{2,6}$, $U_{4,6}$, F_7^-, $(F_7^-)^*$, and P_6, where F_7^- is the non-Fano matroid and P_6 is the 6-element rank-3 self-dual matroid for which a Euclidean representation is shown in Fig. 1.

Let P_8 be the matroid induced by linear independence on the set of columns of the following matrix over $GF(3)$.

$$\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$
$$\left[\begin{array}{c|cccc} & 0 & 1 & 1 & -1 \\ & 1 & 0 & 1 & 1 \\ I_4 & 1 & 1 & 0 & 1 \\ & -1 & 1 & 1 & 0 \end{array}\right].$$

It is not difficult to check, using, for example, a list of its 4-circuits, that P_8 has a transitive automorphism group. Thus every single-element contraction of P_8 is isomorphic to $P_8/1$. The latter is isomorphic to P_7, the matroid for which a Euclidean representation is shown in Fig. 2. Now P_7 is easily shown to be representable over every field other than $GF(2)$ [3]. Using this and the fact that P_8 is self-dual, we deduce that no proper minor

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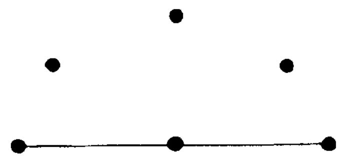

FIGURE 1

of P_8 is a counterexample to Kahn and Seymour's conjecture. The next result completes the argument that P_8 itself is a counterexample.

THEOREM. *P_8 is representable over a field F if and only if the characteristic of F is not two.*

Proof. Suppose that A is a 4×8 matrix representing P_8 over a field F. Then, using Brylawski and Lucas's theory of coordinatizing paths [1], we can assume that

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & \quad 5 & 6 & 7 & 8 \end{matrix} \\ \left[\begin{array}{c|cccc} & 0 & 1 & 1 & d \\ I_4 & 1 & 0 & 1 & 1 \\ & a & 1 & 0 & e \\ & b & 1 & c & 0 \end{array}\right] \end{array}$$

where a, b, c, d, and e are non-zero elements of F. It is straightforward to show, by considering P_8/i for each i in $\{4, 3, 2\}$, that

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & \quad 5 & 6 & 7 & 8 \end{matrix} \\ \left[\begin{array}{c|cccc} & 0 & 1 & 1 & 2 \\ I_4 & 1 & 0 & 1 & 1 \\ & 1 & 1 & 0 & 1 \\ & 2 & 1 & 1 & 0 \end{array}\right] \end{array}.$$

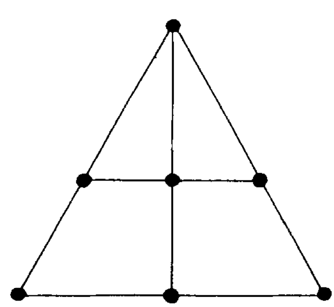

FIGURE 2

Since every non-zero entry in this matrix must be non-zero modulo F, we deduce that F does not have characteristic two. It is now routine to check that A represents P_8 over all fields of characteristic other than two. ∎

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