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SINGLE AND MULTIPARAMETER ESTIMATION USING MULTIMODE INTERFEROMETRY AND SINGLE PHOTONS

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SINGLE AND MULTIPARAMETER ESTIMATION USING MULTIMODE INTERFEROMETRY AND SINGLE PHOTONS

by

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Abstract

This thesis focuses on the utilization of boson-sampling inspired interferometry for the purposes of achieving post-classical sensitivity in single- and multiparameter estimation. Such interferometry architectures take advantage of passive linear optics and single photon inputs whereas alternative schemes can call for nonlinear processes and difficult to construct states. Additionally, the particular architecture used allows for an analytic expression for the permanent of our matrix, generally a classically intractable problem.

In the second chapter, we discuss previously proposed interferometry schemes and then proceed to develop a more optimal architecture based on such a setup. The choice of the quantum Fourier transform as the unitary is examined and compared to alternative special unitary choices including randomly generated special unitaries. Additionally, we study various distributions of phase-generating resources. From this, we determine the necessity of "uniform" unitaries and that phase sensitivity is greatest when all phase-generating resources are grouped within one arm of the interferometer. Using our Quantum Unitary Matrix Interferometer (QUMI), we achieve sub-shot-noise sensitivity. We also obtain an analytic expression for the permanent of the system, which allows for simplified probability calculations.

In the third chapter, we investigate using a similar interferometer for multiparameter estimation. We compare the performance of a parallel quantum Fourier transform interferometer (parallel QuFTI) to repeated measurements with a QUMI and a classical coherent state. We determine that phase sensitivity for the parallel QuFTI, given a detection scheme involving all number-resolving detectors or a single number-resolving detector with the rest on-off detectors, shows exponential improvement over the coherent state and sequential QUMI even at large mode numbers. Additionally, we investigate the interferometer's performance under

nondeterministic photon production and find that the parallel QuFTI outperforms the classical coherent state for as low as 65% fidelity.

Chapter 1: Introduction

1. Introduction to Boson Sampling

Interest in quantum computing is rising among theoretical scientists and physicists alike with many searching for post-classical devices capable of showcasing quantum supremacy. Many examples of such devices [1,2], however, have been deemed practically infeasible due to the large experimental overhead required [3,4]. More recently, Boson Sampling has garnered interest as a more practical method to achieve post-classical results [5, 6].

Boson Sampling schemes make use of passive linear optics, and such networks can generate complicated number-mode entanglement. In Boson Sampling, a passive linear optical network, represented mathematically by a unitary matrix, is fed with uncorrelated single photons. From this input, the output probabilities are given by complex matrix permanents which are of #P-complete complexity class and classically intractable to estimate accurately [7,8].

To better understand Boson Sampling, it can be beneficial to approach the concept through the related notions of quantum and classical random walks. In classical random walks, the position of a walker can be analogous to flipping a coin and proceeding left if the result is heads and right if the result is tails. The probability distribution of position of a random walker—or 'coin'—after a number of steps, *d*, is given by a Gaussian (Fig 1).

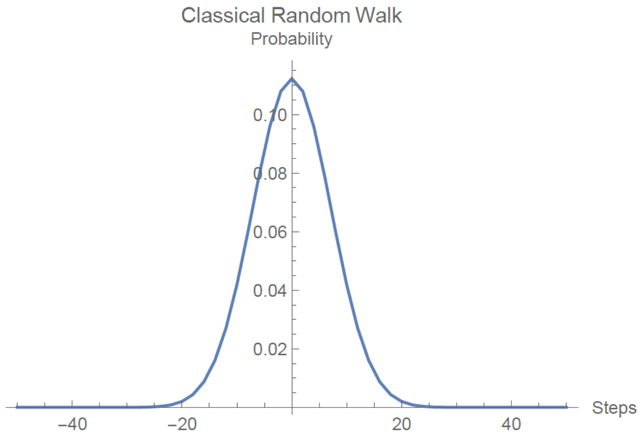

Fig 1: Classical random walk plotted for 50 steps. The classical distribution takes the form of a Gaussian, whereas the quantum examples do not.

At each point in a classical random walk, a step to the left or right is equally likely. Compare this to a quantum random walk (Fig 2) where the initial orientation of the coin can bias the end probability distribution. Additionally, given a coin existing in an unknown 'superposition', the probability peaks towards the outer ends of the system in direct contrast to the classical Gaussian distribution (Fig 3). This occurs due to the quantum coin's ability to interfere with itself along various possible paths. For our purposes, the coin in this case can be considered to be a single photon.

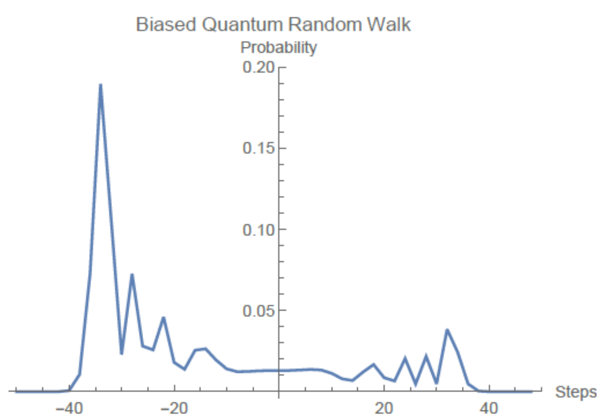

Fig 2: Quantum random walk plotted for 50 steps using an initial state in a biased initial position of "heads". All odd positions have zero probability and are not plotted.

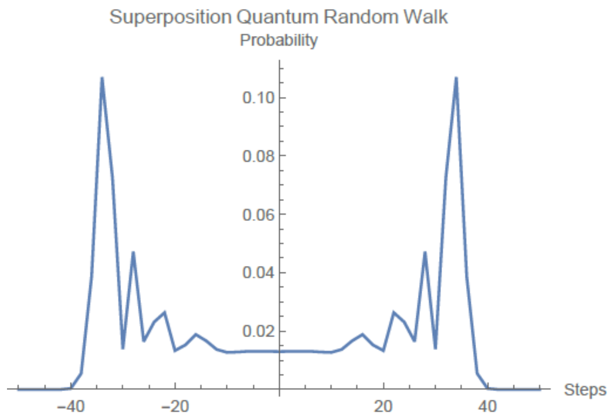

Fig 3: Quantum random walk plotted for 50 steps using an initial state in a initial superposition state. All odd positions have zero probability and are not plotted.

The analogy to Boson Sampling arises if instead there is a system of many quantum walkers progressing through an optical network. If the network is an *n*-dimensional square unitary matrix fed an input of *n* single photons, what will the output distribution look like? The answer turns out to be rather complicated—in fact, it is of #P complexity—and it depends on the matrix in question. The output probability is given by the permanent of the matrix.

Advantages of Boson Sampling include its use of passive linear optics over nonlinear alternatives as well as its use of single photon Fock states rather than more difficult to produce states. It is worth noting that though there are many methods of generating indistinguishable single photons [9-14], all of these methods are still nondeterministic and thus probabilistic photon production can be a challenge.

2. Boson Sampling Applied to Multimode Interferometry

In Ref. [15], Motes *et al*. proposed that an interferometer could use Boson Sampling techniques to achieve nonclassical sensitivity using only a single photon input, on-off detections, and passive linear optics. Their scheme proposed that a network constructed of a unitary matrix, a phase distribution, and the Hermitian conjugate of the first unitary. If the phase distribution is not present, then the network will be equivalent to an identity matrix and the output distribution will match the input distribution. Thus, a small phase distribution could be measured by comparing the output to the identity case.

Similar to Boson Sampling, the aforementioned scheme would use an array of n-single photons sent through a unitary matrix. The probability of achieving either all single photons in the output (*P*) or any other output arrangement (*1-P*) is determined by the permanent of the unitary matrix [15]. If the unitary is constructed to have a simplified equation for the permanent, the unknown phase could be estimated. Thus the principles of Boson Sampling could be used for the purpose of interferometry.

The following two chapters discuss applications of Boson Sampling-inspired interferometers. Additionally, choices of unitary and phase distribution within the architecture are discussed and motivated beyond previous work.

Chapter 2: Single Parameter Estimation

1. Introduction

Previously, it was shown that devices utilizing quantum metrology schemes can result in quantum advantages over classical devices of the same nature through the application of effects such as the Hong-Ou-Mandel effect or the use of NOON states to achieve higher sensitivity [1,2]. Furthermore, devices using Boson Sampling [5,6] have garnered interest due to the increased practicality of using passive linear optic networks over the aforementioned methods. One particular device, shown in Motes *et al.* [15], inspired by these systems has shown to have superior sensitivity to analogous classical devices. We generalize the structure of this device, maximize the phase sensitivity for realistic conditions, and determine not only a more practical scheme but one with superior sensitivity and an analytic form to compute this sensitivity; previously, such a result was merely postulated.

2. Methods

We generalize the architecture developed in [15] to be of the form shown in Fig. 4. The original scheme proposed there can be recreated when $\hat{V}$ is the n-mode quantum Fourier transform (QFT) and $\hat{\Phi}$ is the linear gradient $f_i(\varphi) = (i-1)\varphi$ where i iterates from 1 to n, the dimension of the matrix. Any such device with a fixed unitary of the QFT as in this case will be hereafter referred to as a Quantum Fourier Transform Interferometer (QuFTI).

Table 1: Phase Strategies

Constant	$f_j^{con} = \frac{1}{n}$
Sub-linear	$f_j^{sub} = \sqrt{j}$
Linear	$f_j^{lin} = j$
Quadratic	$f_j^{quad} = j^2$
Exponential	$f_j^{exp} = 2^j$
Delta	$f_j^{\delta} = \delta_{j1}$

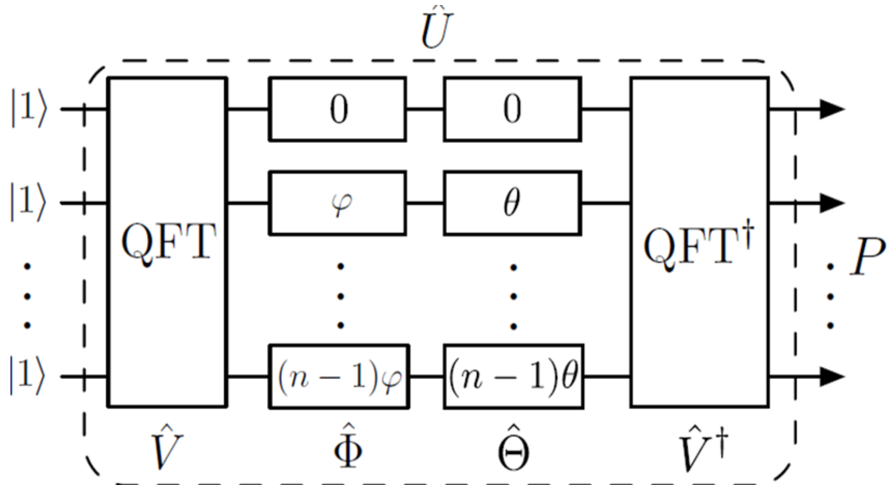

Fig. 4: General QuFTI Architecture
A generalized architecture for the QuFTI, where we optimize for $\hat{V} \in$ SU(n), phase strategy $\widehat{\Phi}$ given single photon inputs and photodetection in each mode.

Using this general architecture, we examine various phase strategies as shown in Table 1 as well as various unitary matrices. We then compute the probability (P) of observing one photon in each mode—this is the observable $\langle\hat{O}\rangle$ used to estimate φ. Using the result of Ref. [7], we can thus compute the probability using the permanent as P = $|perm(\widehat{U})|^2 = |perm(\hat{V}\Phi\hat{V}^\dagger)|^2$. Given that where $u_{i,\sigma(i)}$ denotes the matrix elements of $\widehat{U}$, we find the permanent and phase sensitivity as shown in Eqs. (1,2), respectively.

$$perm(\widehat{U}) = \sum_{\sigma\,\epsilon\,S_n} \prod_{i=1}^{n} u_{i,\sigma(i)}\ , \qquad (1)$$

$$\Delta\varphi = \frac{\sqrt{\langle\widehat{O^2}\rangle - \langle\hat{O}\rangle^2}}{\sqrt{n}\cdot\left|\frac{d\langle\hat{O}\rangle}{d\varphi}\right|}. \qquad (2)$$

In order to motivate the choice of the quantum Fourier transform as the unitary, we generate a large number of random special unitary matrices and select for those that minimize over the phase sensitivity achieved.

3. Results

From the aforementioned procedure, we determine that creating a wider "phase gap" between modes for the QuFTI improves sensitivity (Fig. 5). It is further shown that when compared to the generation of over 10,000 randomly generated special unitary matrices or their minimization, the QFT is optimal (Fig. 6).

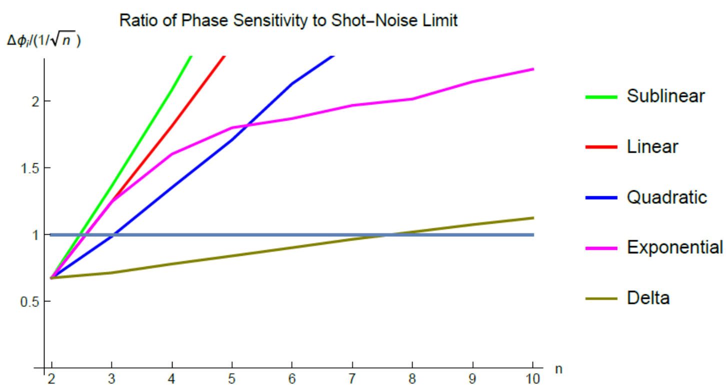

Figure 5: Phase Sensitivity of Varying Strategies
This figure depicts the ratio of phase sensitivity to the Shot-Noise limit. The shot-noise limit used here is $\frac{1}{\sqrt{n}}$. Anything below 1 indicated sub-shot-noise phase sensitivity.

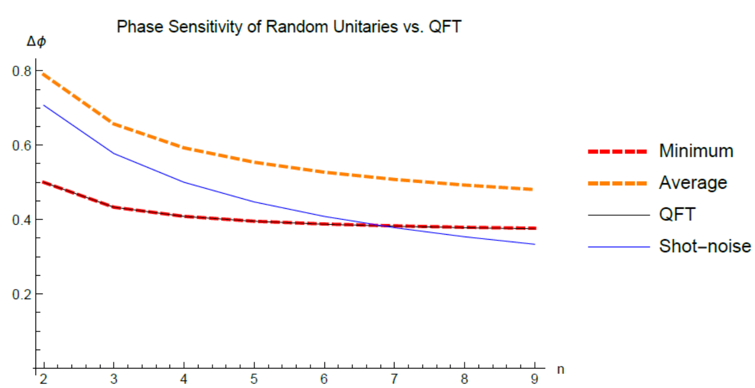

Figure 6: QFT is Optimal for Delta Strategies
This figure depicts the phase sensitivity of random unitaries as well as that of QuFTI. For $2 \leq n \leq 7$, the sensitivity of the QFT exceeds that of every random unitary.

Using the "delta" phase strategy, we are able to determine an analytical permanent for the QuFTI as shown here:

$$perm(\hat{U}) = \frac{1}{n^n}\sum_{k=0}^{n} D_{n,k}[e^{i\varphi} + n - 1]^k[e^{i\varphi} - 1]^{n-k}\,, \quad \textbf{(3)}$$

where:

$$D_{n,k} = \frac{n!}{k!}\sum_{j=0}^{n-k}\frac{(-1)^j}{j!}. \quad (4)$$

If we consider the Quantum Fisher Information for a delta phase strategy where the phase shift is place in the first mode with an input initial state of single photons $|1\rangle^{\otimes m}$, we explicitly compute it to be $F(|\psi_\varphi\rangle) = 8(1 - \sum_{i=1}^{n}|V_i|^4)$, where V_i is the *i*-th element in the top row of $\hat{V}$. This calculation is shown in Ref. [16]. This means that the Fisher information is dependent only on the coupling between the input modes and the first output mode.

We also determine that while the QFT is optimal, it is not uniquely so. By computing the Quantum Fisher Information when *k* photons $|k\rangle^{\otimes m}$ are fed into each mode of $\hat{V}$ to be:

$$F(|\psi_\varphi\rangle) = 4(1 - \sum_{i=1}^{n}|V_i|^4)k(k+1) \quad (5)$$

Which is maximized for $\hat{V}$ with $|\boldsymbol{V_i}| = \frac{1}{\sqrt{n}}$ for all elements on the top row. The QFT does satisfy this constraint; however, many other matrices do as well. We consider any unitary with this structure to be "uniform" and any interferometer with these unitaries to be a quantum unitary multimode interferometer (QUMI). A QuFTI is therefore a special case of a QUMI. Due to the phase sensitivity relying solely on the values of the first row of the matrix, a network can attain the maximum sensitivity with only *O(n)* beam splitters; this is a significant improvement over the scheme proposed in Motes *et al.* which requires $O(n^2)$ beam splitters [15,16]. A simple implementation is shown in Fig 7, where the transmittivity of the beam splitter acting on modes 1 and *k* should be $\frac{1}{\sqrt{k}}$. Setting $|V_i| = \frac{1}{\sqrt{n}}$ gives:

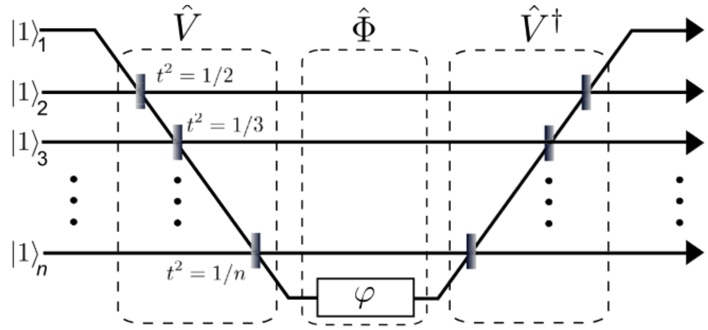

FIG. 7. Simple architecture that maximizes the phase sensitivity of our scheme. Subscripts on the kets denote mode number. The beam splitters (vertical gray slabs) should be adjusted so that $\hat{V}$ is a QUMI; namely, the transmittivity amplitude t of the beam splitter acting on mode 1 and mode k should be $\frac{1}{\sqrt{k}}$.

4. Conclusion

After consideration of various phase gradients and unitary schemes, we developed an optimal architecture for an interferometer, consisting of an implementation of an n-mode QFT, all phase resources in a single arm, and finally the Hermitian conjugate of the QFT. For small n, $n < 7$, the sensitivity of the QuFTI is sub-shotnoise. The structure of QuFTI uses only passive linear optics and single-photon sources, meaning the implementation is far more convenient than alternative schemes, and it can be further simplified to the structure of a QUMI. Furthermore, the similarity to Boson Sampling architecture implies that the implementation of the QUMI is already achievable with current technology.

Chapter 3: Multiparameter Estimation

1. Introduction

It was previously shown that optical networks using only single photon Fock states, passive linear optics, and single photon detection could achieve post-classical sensitivity for single-parameter estimation when the mode number is small [15,16]. Furthermore, it was shown that multiparameter estimation could achieve similar results with number-resolving detection [17]. We consider an analogous architecture to the Quantum Fourier Transform Interferometer(QuFTI) proposed in Ref. [15] for the estimation of multiple phases simultaneously. Thus, we will see that this yields post-classical sensitivity for multiparameter estimation even for an asymptotically large number of modes. This system is also considered for nondeterministic photon sources and a variety of measurement schemes.

2. Multiparameter Estimation in a Parallel QuFTI

We consider an analogous architecture to the QuFTI proposed in Ref. [15,16], with single photon inputs in each mode, but instead of a single phase, we estimate multiple independent phases simultaneously. The interferometer consists of m modes with a photon in each mode with input $|\psi_{in}\rangle = |1\rangle^{\otimes m}$, as shown in Fig. 8.

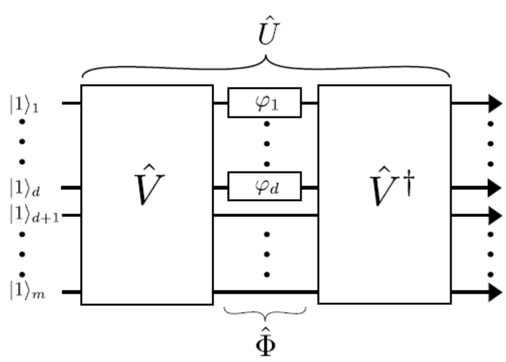

Fig 8: Architecture of proposed parallel QuFTI optical interferometer, which simultaneously measure *d* independent phases.

The input is fed into a passive linear optical unitary, given by $\hat{U} = \hat{V}\hat{\Phi}\widehat{V^{\dagger}}$, with $V = \{V_{ij}\}$ being the quantum Fourier transform and $\hat{\Phi} = \{\Phi_{kl}\}$ being a diagonal array of *d* independent phases $\vec{\varphi} = \{\varphi_j\}_{j=1}^{d}$.

$$\hat{V} = \frac{1}{\sqrt{m}} e^{2\pi(i-1)(j-1)/m}, \tag{5}$$

$$\Phi_{kl} = \begin{cases} \delta_{kl} \cdot e^{i\varphi_k} & for\ k \leq d \\ \delta_{kl} & for\ k > d \end{cases}. \tag{6}$$

Other than the form of $\hat{\Phi}$, the above is identical to our previous QuFTI of Fig 3, which leads us to refer to this device as a “parallel QuFTI”. In following sections, we consider several different measurements ranging from photon counting with number-resolution to on-off photodetection which only distinguishes zero from a non-zero number of photons. The resulting probability distribution for each strategy is obtained from repeated measurements then acts as a measure of the unknown phases.

The quantum Cramér-Rao bound (QCRB) limits the uncertainty in our measurement and in this case is given by:

$$|\Delta\varphi|^2 \geq \frac{d(m-d+1)}{8v\,(m-d)}. \tag{7}$$

where $|\Delta\varphi|^2$ is the variance, *d* the number of phases, *m* the number of modes, and *v* the number of trials. We thus consider the viability of the various measurement schemes by comparing them to this bound.

3. Measurement Strategies

To demonstrate the benefit of our system, we consider additional metrology setups where all are restricted to use the same number of photon resources (Fig. 9). The classical case is represented by an uncorrelated coherent state. We also consider a repeated use of a quantum unitary interferometer (Sequential QUMI).

Previously, the architectures proposed for single parameter estimation required only on-off photon detection [15,16]; however, it is uncertain that that would be sufficient for multiparameter estimation. Thus, we compare not only to the aforementioned interferometry architecture but also to the sensitivity achieved using a variety of detection schemes with the parallel QuFTI. We consider detection with all single photon detectors (SPD), all number resolving detectors (All NRD), and a scheme with one number resolving detector with the remaining detectors being single photon detectors (One NRD) against the QCRB for the parallel QuFTI architecture.

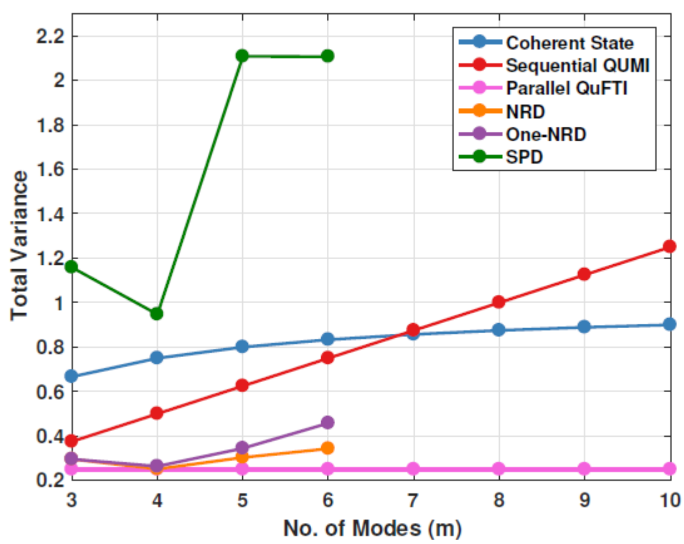

Fig 9: Comparison of variance of Measurement Strategies showing an asymptotic improvement for the Parallel QuFTI over the Coherent State. This is nearly achieved for NRD and One-NRD.

When considering the QCRB of the system, the parallel QuFTI shows an asymptotic improvement over the classical coherent state. This theoretical bound is nearly achieved using NRD and One-NRD detection schemes. However, the scheme using all on-off detection is unable to perform comparably for multiparameter estimation. Thus at least one number-resolving detector would be necessary for increased phase sensitivity.

4. Probabilistic Photon Sources

Although single-photon production has greatly advanced in recent years, many production techniques such as spontaneous parametric down conversion (SPDC) are probabilistic [9-14]. Thus, an input state consisting of *m* photons is not always guaranteed. For this reason, we consider a “scattershot” input similar to Ref. [14,18] to account for probabilistic photon generation. In a manner analogous to those previous proposed, we show that our scheme can still provide a sub-shot-noise sensitivity even when photon sources are not necessarily reliable on-demand sources.

In this scattershot approach, there is a source in each input mode that generates a photon pair, such as with SPDC. One photon enters the interferometer while the other is detected to herald its twin. This allows us to keep track of which mode of the interferometer has a photon as well as how many photons total are within the system. We consider each source has probability ($p_i < 1$) of generating a photon.

To calculate the variance, consider *m* SPDC photon sources where the probability of generating a particular input configuration is p_i. For each configuration, the associated variance $\Delta\vec{\varphi}_i^{\,2}$ is calculated from the classical Fisher information, and the average variance $\Delta\vec{\varphi}^2_{avg}$ is given by:

$$\Delta\vec{\varphi}_{avg}^{\,-2} = \sum_{i=1} p_i \, \Delta\vec{\varphi}_i^{\,-2} \,, \qquad (8)$$

where the summation is over the total number of input configuration. In the following figure, all sources are given equal probability *p* of emitting a heralded photon for simplicity.

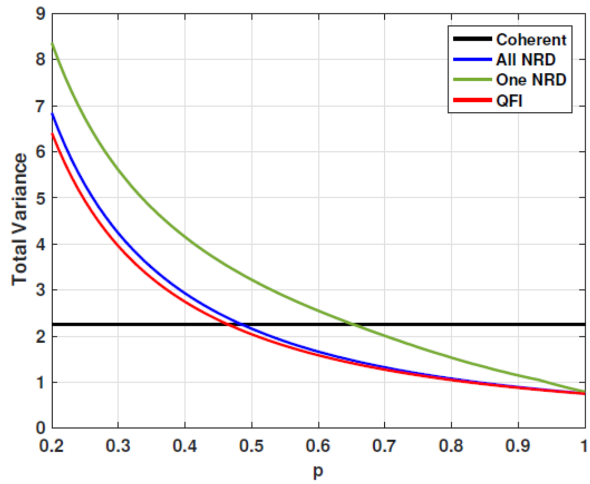

Figure 10: 4-arm, 3-phase example of variance versus photon production probability, assuming all modes have the same probability *p*. The detection schemes presented are the same as those in Fig. 9

We numerically consider a 4 mode, 3 phase Parallel QuFTI which—as seen above—can still outperform a lossless coherent state at around 50% efficiency, or around 65% efficiency for a one-NRD detection scheme.

5. Conclusion

We have shown that a passive, multi-mode interferometer used for multiparameter estimation can demonstrate high sensitivity. Its quantum Cramér-Rao bound shows an asymptotic improvement by a factor of four over classical schemes and can achieve this with relatively simple setups and inefficient but heralded single-photon sources.

As the number of modes increases, we expect that a single NRD will be insufficient to capture the required information to allow the device to be supersensitive. A future analysis of the scaling of the NRDs required would be useful to determine the scalability of this device.

Chapter 4: Conclusions and Summary

Throughout this thesis, we demonstrate the applicability of boson-sampling-inspired interferometry to single- and multiparameter estimation.

In Chapter 1, we began with an introduction to Boson Sampling and its application to multimode interferometry. We discussed the concepts of Boson Sampling in terms of quantum and classical random walks. We then discussed the previously proposed methods of applying Boson Sampling to an interferometer formed of a simple unitary structure to obtain post classical sensitivity as well as some of the issues that needed to be addressed with that architecture.

In Chapter 2, we motivated the choices of the aforementioned interferometry design by investigating optimal unitaries and phase gradients to be employed for this purpose. We determine that placing all phase generating resources in a single arm of the interferometer yields the greatest phase sensitivity. By comparing the quantum Fourier transform to minimized special unitaries, we find that it is optimal for our purposes although not uniquely so. After computing the quantum Fisher information given the delta phase distribution, we find that any "uniform" unitary will perform optimally. Additionally, the phase distribution depends solely on the first-row elements; thus, the interferometer can be further simplified to scale as $O(n)$ beam splitters. We show that this new architecture achieves sub-shot-noise sensitivity for $n < 7$. Additionally, we compute an analytic permanent of this system.

In Chapter 3, we generalize the QuFTI architecture for the purposes of multiparameter estimation. We calculate the quantum Cramér-Rao bound of the set up and then compare the parallel QuFTI's performance to that of a classical coherent state as well as repeated phase estimation with a QUMI. The QuFTI demonstrates asymptotic improvement by a factor of four over the classical state even for large *n.* Additionally, we compare the performance of the multimode interferometer, given different detection schemes. We show that for detection with all number-resolving detectors or a single number-resolving detector, phase sensitivity approaches the theoretical bound.

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Vita

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