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The effects of quantum field renormalization on the predictions of inflation for the CMB anisotropies

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Abstract. In single-field, slow-roll inflationary models scalar and tensorial (Gaussian) perturbations are usually characterized by the so called power spectrum in momentum space. Even though these power spectra are finite and well define in momentum space, typical ultraviolet divergences in quantum field theory appear when these quantities are expressed in position space. The requirement of a finite variance in position space forces the introduction of regularization technics in quantum field theory in an expanding universe. The regularization process has an important impact on the predicted scalar and tensorial power spectra for wavelengths that today are at observable scales.

1. Introduction

Inflation nowadays constitutes an important piece of the standard cosmological model [1]. It was originally introduced as a mechanism able to alleviate the classical problems of the big bang model [2]. However, it was soon realized [3] that inflation is able to provide an elegant quantum mechanical mechanism to account for the origin of small inhomogeneities in the early universe, essential for explaining the structures that we see today. In the simplest models of inflation, a scalar field slowly rolling down its potential causes an accelerated expansion of the background geometry. The rapidly changing spacetime geometry is able to amplify the vacuum fluctuations [4] of the inflaton field itself and of purely tensorial modes (gravitational waves), generating a primordial spectrum of scalar and tensor perturbations. These perturbations acquire classical properties during their evolution and provide the initial conditions for classical cosmological perturbations. The potential detection of the effects of primordial tensorial metric fluctuations in future high-precision measurements of the CMB anisotropies would constitute a highly non-trivial test of the inflationary paradigm. Therefore, it is particularly important to scrutinize from all point of views the predictions of inflation for the tensorial and scalar power spectra. In this respect, it was pointed out in [5] (see also [6]) that quantum field renormalization significantly modifies the amplitude of quantum fluctuations, and hence the corresponding power spectra, in de Sitter inflation. The analysis was further improved in [7, 8] to understand how the basic testable predictions of (single-field) slow-roll inflation could be affected by renormalization. In this talk we summarize our approach.

Let us assume that $\varphi(\vec{x}, t)$ represents a perturbation obeying a free field wave-equation on the inflationary background $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$, where $a(t)$ is a quasi-exponential expansion factor. At the quantum level, this field is expanded as $\varphi(\vec{x},t) = (2\pi)^{-3/2}\int d^3k[\varphi_k(t)a_{\vec{k}} + \varphi_k^*(t)a_{\vec{k}}^\dagger]e^{i\vec{k}\vec{x}}$, where the creation and annihilation operators satisfy the canonical commutation relation. The mode functions $\varphi_k(t)$ are required to satisfy the adiabatic condition [9]. The power spectrum for this perturbation, $\Delta_\varphi^2(k,t)$, is usually defined in terms of the Fourier transform of the variance of the field [1]

$$\langle\hat{\varphi}_{\vec{k}}(t)\hat{\varphi}_{\vec{k}'}^\dagger(t)\rangle = \delta^3(\vec{k}+\vec{k}')\frac{2\pi^2}{k^3}\Delta_\varphi^2(k,t) \ , \tag{1}$$

where $\hat{\varphi}_{\vec{k}}(t) \equiv \varphi_k(t)a_{\vec{k}}$. The field modes $\varphi_k(t)$ describe a perturbation field characterized, in momentum space, by a zero mean $\langle\hat{\varphi}_{\vec{k}}(t)\rangle = 0$ and the variance $\Delta_\varphi^2(k,t)$. The advantage of working in momentum space resides in the fact that different modes fluctuate independently of each other, as explicitly displayed by the presence of the delta function in (1). This way, the quantum field is regarded as an infinite collection of independent oscillators, each with a different value of $\vec{k}$. In position space the perturbation is also characterized by a zero mean $\langle\varphi(\vec{x},t)\rangle = 0$ and a variance $\langle\varphi^2(\vec{x},t)\rangle = (2\pi)^{-3}\int d^3kd^3k'\langle\hat{\varphi}_{\vec{k}}(t)\hat{\varphi}_{\vec{k}'}^\dagger(t)\rangle e^{i(\vec{k}+\vec{k}')\vec{x}}$. This variance is formally related to the power spectrum by

$$\langle\varphi^2(\vec{x},t)\rangle = \int_0^\infty \frac{dk}{k}\Delta_\varphi^2(k,t) \sim \frac{1}{4\pi^2}\int_0^\infty dk(\frac{k}{a^2}+\frac{\dot{a}^2}{a^2k}+...) \ , \tag{2}$$

where the large k behavior of the integrand is shown. As it is well-known in quantum field theory, the above expectation value quadratic in the field φ is ultraviolet divergent (quadratic and logarithmically). The quadratic divergence corresponds to the usual contribution from vacuum fluctuations in Minkowski space and can be eliminated by standard renormalization in flat spacetime. The logarithmic divergence, however, appears as a consequence of the non trivial spacetime expansion. Because the different k-modes fluctuate independently of each other, one could be tempted to get rid of this logarithmic ultraviolet divergence by simply eliminating the modes with $k > aH$ and leaving the rest unaffected. If one eliminates this divergence using a window function in this way, as is usual for random fields, then one obtains $\Delta_\varphi^2(k) \approx H^2/4\pi^2$, where $\Delta_\varphi^2(k)$ is defined by the quantity $\Delta_\varphi^2(k,t)$ evaluated a few Hubble times after the "Hubble exit time" t_k (defined by $a(t_k)/k = H(t_k)$), since this is the time scale at which the modes become time independent and behave as classical perturbations. However, one should take into account that the field fluctuations are quantum in nature and, therefore, one should consider the subtle points of quantum field theory (QFT) regarding the ultraviolet divergences.

Even though free quantum field theory is usually regarded as an infinite set of independent harmonic oscillators (one for each k-mode), there are fundamental holistic aspects of QFT that can not be properly understood in terms of independent modes. Renormalization is the hallmark of the holistic aspect of QFT [10]. This is clear in the fact that, although the renormalization schemes in QFT in curved spacetimes are based on the ultraviolet behavior of the theory, the infrared sector is also affected by renormalization, leading potentially to observable consequences. Therefore, the logarithmic divergence in (2) should be dealt with by renormalization in curved spacetimes. We propose that in the standard definitions of the spectrum $\Delta_\varphi^2(k,t)$, as given in (2), one should replace the unrenormalized $\langle\varphi^2(\vec{x},t)\rangle$ by the renormalized variance, $\langle\varphi^2(\vec{x},t)\rangle_{ren}$. Writing $\tilde{\Delta}_\varphi^2(k,t)$ for the spectrum defined in this way, the definition in (2) is replaced by the corresponding renormalized expression

$$\langle\varphi^2(\vec{x},t)\rangle_{ren} = \int_0^\infty \frac{dk}{k}\tilde{\Delta}_\varphi^2(k,t) \ . \tag{3}$$

Since the power spectrum is defined in momentum space, the natural scheme is renormalization

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in momentum space, so we define

$$\langle\varphi^2(\vec{x},t)\rangle_{ren} = \frac{4\pi}{(2\pi)^3}\int_0^\infty k^2 dk(|\varphi_k(t)|^2 - C_k(t)) \ , \tag{4}$$

where $C_k(t)$ represents the renormalization subtraction terms. Adiabatic renormalization [11, 9] provides a natural expression for $C_k(t)$. Moreover, the Bunch-Parker renormalization in momentum space [12, 9] (which turns out to be equivalent, when translated to position space, to the DeWitt-Schwinger proper time renormalization) provides another answer for $C_k(t)$. When these schemes are applied to the field perturbations arising from slow-roll inflation, which should be considered as massless free fields with a second-order adiabatic term encoding the dependence on the inflationary potential, the resulting expressions for $C_k(t)$ coincide, thus defining a unique expression for the spectrum $\tilde{\Delta}^2_\varphi(k,t)$. The holistic nature of QFT is then explicitly realized through (3-4); although the counterterms are fully determined by the ultraviolet behavior of the modes, the long wavelength sector, and hence the new $\tilde{\Delta}^2_\varphi(k,t)$, is significantly affected by the subtractions. In the slow-roll scenario, when H slowly decreases with time, the effects of renormalization have a non-trivial impact on $\tilde{\Delta}^2_\varphi(k,t)$ when this quantity is evaluated a few n Hubble times after the Hubble exit time t_k ($n > 1$ but $n\epsilon \ll 1$). We obtain for the tensorial and scalar spectra

$$\begin{aligned}\tilde{\Delta}^2_t(k,n) &\approx \frac{8}{M_P^2}\left(\frac{H(t_k)}{2\pi}\right)^2 \epsilon(t_k)(2n-3/2)\\ \tilde{\Delta}^2_s(k,n) &\approx \frac{1}{2M_P^2\epsilon(t_k)}\left(\frac{H(t_k)}{2\pi}\right)^2 (3\epsilon(t_k)-\eta(t_k))(2n-3/2) \ ,\end{aligned} \tag{5}$$

where $\epsilon = M_P^2/2(V'/V)^2$ and $\eta = M_P(V''/V)$ are the standard slow-roll parameters associated with the inflaton potential energy V. It is important to note that the subtraction terms introduce a time dependence, characterized by n, in the power spectra parameterizing the (unknown) time at which the modes exhibit classical behavior. However, the spectral indices and the tensor-to-scalar ratio r are not sensitive to the unknown parameter n, in the same way as they do not depend on the scale of inflation $H(t_k)$

$$\begin{gathered}\tilde{n}_s - 1 \equiv \frac{d\ln\tilde{\Delta}^2_s}{d\ln k} = -6\epsilon + 2\eta + \frac{(12\epsilon^2 - 8\epsilon\eta + \xi)}{3\epsilon - \eta} \quad ; \quad \tilde{n}_t \equiv \frac{d\ln\tilde{\Delta}^2_t}{d\ln k} = 2(\epsilon-\eta)\\ \tilde{r} \equiv \frac{\tilde{\Delta}^2_t}{\tilde{\Delta}^2_s} = 16\epsilon\frac{\epsilon}{3\epsilon-\eta} \ ,\end{gathered} \tag{6}$$

where ξ is another slow roll parameter: $\xi \equiv M_P^4(V'V'''/V^2)$. This parameter can be reexpressed in terms of ϵ, η and the running of the tensorial index $\tilde{n}'_t \equiv d\tilde{n}_t/d\ln k$ as $\tilde{n}'_t = 8\epsilon(\epsilon-\eta) + 2\xi$.

The expression (5) and (6) for the power spectra and spectral indices show that renormalization has a significant impact on the predictions of slow-roll inflation. It is important to notice that, in contrast to the standard approach, the renormalized tensorial power spectrum is not uniquely related to the Hubble parameter and now depends on the combination $H(t_k)/\epsilon(t_k)$. Another important difference concerns the consistency relation that relates the tensor-to-scalar ratio r to the spectra indices. This relation, that reads $r \equiv \Delta^2_t/\Delta^2_s = -8n_t$ if renormalization is not considered, becomes now

$$\tilde{r} = 4(1-\tilde{n}_s-\tilde{n}_t) + \frac{4\tilde{n}'_t}{\tilde{n}_t^2 - 2\tilde{n}'_t}\left(1-\tilde{n}_s - \sqrt{2\tilde{n}'_t + (1-\tilde{n}_s)^2 - \tilde{n}_t^2}\right) \ . \tag{7}$$

This expression is much more involved that the standard one allowing a wider range of possibilities. For instance, an exact scale-invariant tensorial power spectrum, $\tilde{n}_t = 0$, is now compatible with a non-zero ratio $r \approx 0.12 \pm 0.06$, which is forbidden by the standard prediction. Also, the values of $\tilde{n}_t$ are not constrained to be negative in contrast with the unrenormalized result. Additionally, the running of $\tilde{n}_t$ can not be determine form the values of n_s and n_t, and it is now an independent quantity that needs to be measured independently. This aspect could make more challenging the experimental verification of the consistency condition of (single-field) slow roll inflation.
An important question is related to the time at which the subtraction terms employed in the renormalization procedures should be evaluated. A definitive answer requires a deep analysis of the process in which the quantum perturbation acquire classical properties and its relation with quantum field renormalization. However, the fact that we find observable differences offers a deep way to experimentally probe this question.

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