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**ERRATUM: ABSOLUTE CONTINUITY OF LAWS FOR
 SEMILINEAR STOCHASTIC EQUATIONS WITH
 ADDITIVE NOISE
 (COSA, VOL. 2, NO. 2 (2008) 209–227)**

BENEDETTA FERRARIO

On page 212 of [1], the proof of Theorem 2.1 is wrong at equation (2.3). Different assumptions are required:

- [A1] given $x \in E$, on the time interval $[0, T]$ equation (1.1) has a weak solution $((\Omega, \mathbb{F}, \{\mathbb{F}_t\}_{t \in [0, T]}, \mathbb{P}), w, u)$ with initial data x and a. e. path $u \in C([0, T]; E)$;
- [A2] given any $y \in E$, on any time interval $[t_0, T] \subseteq [0, T]$ equation (1.2) has a unique strong solution z with initial data y and a. e. path $z \in C([t_0, T]; E)$.

Here are the corrected statements. For the proofs, see [2].

Theorem 2.1. *Assume [A1], [A2], and on the time interval $[0, T]$ consider the solutions u and z of equations (1.1) and (1.2) respectively, with the same initial data $x \in E$.*

If

$$\mathbb{P}\left\{\int_0^T |G^{-1}F(z(s))|_H^2 ds < \infty\right\} = 1, \quad (2.1)$$

$$\mathbb{P}\left\{\int_0^T |G^{-1}F(u(s))|_H^2 ds < \infty\right\} = 1, \quad (2.1')$$

then

(i) *the stochastic process*

$$\rho_t(z) = e^{-\int_0^t \langle G^{-1}F(z(s)), dw(s) \rangle_H - \frac{1}{2} \int_0^t |G^{-1}F(z(s))|_H^2 ds}, \quad 0 \leq t \leq T,$$

is a positive $\{\mathbb{F}_t\}$ -martingale; in particular, $\mathbb{E}[\rho_T(z)] = 1$.

(ii) *the stochastic process*

$$w^*(t) = w(t) + \int_0^t G^{-1}F(z(s)) ds, \quad t \in [0, T], \quad (2.2)$$

is an H -cylindrical Wiener process with respect to \mathbb{P}^ , where the probability measure \mathbb{P}^* is defined on (Ω, \mathbb{F}_T) by*

$$d\mathbb{P}^* = \rho_T(z) d\mathbb{P}.$$

Theorem 2.3. *Assume [A1], [A2], and on the time interval $[0, T]$ consider the solutions u and z of equations (1.1) and (1.2) respectively, with the same initial data $x \in E$. If (2.1) and (2.1') hold, then $\mathcal{L}^F \sim \mathcal{L}^0$ and the Radon-Nykodim derivatives are*

$$\frac{d\mathcal{L}^F}{d\mathcal{L}^0}(z) = \mathbb{E} \left[e^{-\int_0^T \langle G^{-1}F(z(s)), dw(s) \rangle_H - \frac{1}{2} \int_0^T |G^{-1}F(z(s))|_H^2 ds} \middle| \sigma_T(z) \right], \quad \mathbb{P} - a.s. \quad (2.6)$$

$$\frac{d\mathcal{L}^0}{d\mathcal{L}^F}(z) = \mathbb{E}^* \left[e^{+\int_0^T \langle G^{-1}F(z(s)), dw^*(s) \rangle_H - \frac{1}{2} \int_0^T |G^{-1}F(z(s))|_H^2 ds} \middle| \sigma_T(z) \right], \quad \mathbb{P}^* - a.s. \quad (2.7)$$

Moreover, \mathcal{L}^F is unique.

For completeness, we substitute Corollary 2.4 of [1] with the following

Corollary 0.1. *Assume [A1] and [A2]. If*

$$|G^{-1}F(v)|_H \leq c(1 + |v|_E^p), \quad \forall v \in E, \quad (2.8)$$

for some constants $p > 0$ and $c > 0$, then conditions (2.1) and (2.1') are fulfilled and therefore Theorem 2.3 holds true.

The main results of Sections 3 and 4 in [1] remain true. More precisely, from previous results the Kuramoto-Sivashinky equation is known to have a unique strong solution (see the discussion after Theorem 3.4 in [1]). Therefore for Section 3, the only change to do is in Theorem 3.4 of [1]: erase in the second line the sentence "there exist a unique weak solution of equation (3.2) on $[0, T]$. Its". In the same way, change Theorem 4.3 in [1]; for this we need to know that the stochastic hyperviscosity-regularized Navier–Stokes equation has a solution. In [3] it has been proved that for any $\alpha \geq \frac{5}{4}$, given $x \in D(A)$ there exists a unique strong solution u such that $u \in C([0, T]; D(A))$ \mathbb{P} -a.s. By the way, estimate (4.8) in [1] requires Lemma 4.4 in [3].

References

1. Ferrario, B.: Absolute continuity of laws for semilinear stochastic equations with additive noise, *Commun. on Stoch. Anal.* **2** (2008), no.2, 209–227.
2. Ferrario, B.: A note on a result of Liptser-Shiryaev, (2010), e-print arXiv:1005.0237v2, available at <http://arxiv.org/abs/1005.0237>
3. Ferrario, B.: Well posedness of a stochastic hyperviscosity-regularized 3D Navier–Stokes equation, (2010), e-print arXiv:1002.4989v1, available at <http://arxiv.org/abs/1002.4989>

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