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Karl Heinrich Hofmann
Technische Universitat Darmstadt, Germany, hofmann@mathematik.tu-darmstadt.de

TULANE UNIVERSITY
College of Arts and Sciences
NEW ORLEANS, LOUISIANA 70118

Department of Mathematics

Area Code: 504
Telephone: 865-5727
Cable: TULIM

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Prof.J.D. Lawson
Department of Mathematics
LSU
Baton Rouge,La.

Dear Jimmie:
I have several rather postive responses about the special session and the workshop from various peop~~x~~le(W.Taylor,Jane M.Day. Y.W.Lau, R.P.Hunter; K.A. Baker was very helpful but wants to be counted out.) Do we write G. Birkhoff ,too? I have no response yet from Harrold(Assoc.Secr.~~MAS~~ AMS).

On our paper: it is being typed ,but I am troubled by a mistake that crept into a portion that I contributed during one of the later stages. On p. ~~MMM~~ 117 (in 2.22) Sublemma B , proof , I claim that for compact T and discrete J we have $\beta(T \times J) = T \times \beta J$. For infinite J this is ~~x~~ incorrect. I believe that this makes the statement of SUBLEMMA B incorrect,too.

Next this affects Proposition C . ~~Itxxhmmldxxmmdx~~ What is left momentarily is this:
SUBLEMMA C. Let T be a compact chain and J a set. ~~Then~~ Write $Q(J,T) = \Omega_T(\beta J\cancel{\#})$ for convenience. Then $Q(J,T)$ is a quotient of ${}^J T$. Moreover, If $f_j : T \longrightarrow S$ is a family of morphisms , $j \in J$ and $f: {}^J T \longrightarrow S$ the unique coproduct morphism, then f will factor through ${}^J T \longrightarrow Q(J,T)$ iff the function F_o: $\mathrm{cone}_T\, J \longrightarrow S$ given by $F_o([t,j]) = f_j(x)$ extends to a continuous F: $\mathrm{cone}_T\, \beta J \longrightarrow S$.

[Proof of the fact that there is a quotient map $q: {}^J T \longrightarrow Q(J,T)$: By the coproduct property there is a unique q such that $q(\mathrm{copr}_j(t)) = [t,j]$.]

Further , in Proposition D, what remains is

SUBLEMMA D. Let T be a compact chain, Then IRR $Q(J,T)$ = PRIME $Q(J,T)$ is homeomorphic to $\mathrm{cone}_T\, \beta J$, and every generating set contains the image of the map $t \longmapsto [t,j]: T \longrightarrow Q(J,T)$, for each $j \in J$. In particular, the union of these images is the unique smallest generating set..

Sublemma B has got to go or be replaced by something meaningful.
All of this is relatively harmless (outside the ~~x~~ most annoying fact that we still do not know what ${}^J I$ is (since $Q(J,I)$ gives only an inkling of its size), but what is more serious are the potential consequences for the proof of Theorem 4.7, one of the core results of the paper.
The crucial use is made in line 3 ff on ~~px~~ p.22 (your write-up)

I suggest the following measure: Replace the sentence beginning "Now we invoke Proposition D in example 2.22..." by the following: Thus by 4.6 we have $\max\{\mathrm{sdim}\ {}^{A}2,\ \mathrm{sdim}\ {}^{B}I\} \le \mathrm{sdim} \coprod_J T_j$. By 2.22 a) and b) we know that $\mathrm{sdim}\ {}^{A}2 = \mathrm{card}\ A$. By 4.6 again we observe $\mathrm{sdim}\ Q(\mathbb{B}, I) \le \mathrm{sdim}\ {}^{\mathbb{B}}I$ in view of SUBLEMMA C in 2.22. By SUBLEMMA D of 2.22 we obtain ~~know that~~ $\mathrm{sdim}\ Q(\mathbb{B}, I) = \mathrm{card}\ B$. Thus $\max\{\mathrm{card}\ A, \mathrm{card}\ B\} \le \mathrm{sdim} \coprod_J T_j$.

I believe that this will repair the gap. Please check on this and let me know.

The error described above also appears in my SCS memo of 9-20-76 and was brought to my attention by Keimel. (The material of 2.22 was worked out by Gierz, Keimel, Mislove, myself this last summer, but somehow this slipped by us.) The question of copowers is thus still open. (In some sense I am glad that I did not notice this earlier, because then it is not likely that I would have developed the <u>CL</u>-theory of spaces of lower semicontinuous functions contained in that memo. I went through this material in a seminar and so far it appears to hold up (except for a few minor typos and rectifications).

As always

Karl