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Volume 1 | Issue 1 Article 100

10-5-1976

An Error in the Copower Considerations

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~~to~~ Hofmann only

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SEMINAR ON CONTINUITY IN SEMILATTICES (SCS)

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		october 5	76	

TOPIC An Error in the copower considerations

REFERENCE HOFAMNN SCS-Memoir from 9/20/76

In the SCS-Memoir cited above, the following assertion is used in the proof of lemma 2.4:

For every compact space S, $\beta(J\times S) \cong \beta J\times S$.

This is wrong for two reasons: Firstly, in Trans. Amer.Math. Soc. 90 (1959), p.369 ff., Glicksberg has shown that $X\times Y$ is pseudo-compact if and only if $\beta(X\times Y)\cong \beta X\times \beta Y$ (A space is pseudo-compact if every ~~bounded~~ realvalued continuous function on it is bouded),provided that X and Y are both infinite.

More concretely, let us consider the example, where $J = \mathbb{N}$ and $S = I$ the unit interval.

Let $x_n = (1/n, n)$ and $y_n = (0, n)$ for all natural numbers n.

For every free ultrafilter u on $\mathbb{N}$, y_n and x_n converge to the same point $(0,u)$ in $\beta\mathbb{N}\times I$. At the other hand, let $f\colon \mathbb{N}\times I \longrightarrow I$ be defined by $f(n,a) = \min(na,1)$. Then f is continuous on $\mathbb{N}\times I$, and $f(x_n) = 1$, $f(y_n) = 0$ for all n.

Thus, f cannot be extended continuously onto all of $\beta\mathbb{N}\times I$.

This also shows that $\beta(\mathbb{N}\times I) \ncong \beta\mathbb{N}\times I$.

Note, that f is a CL-morphism on all the fibers $\{n\}\times I$. Thus, this example also shows that $\beta\mathbb{N}\times I$ does not generate the copower $^{\mathbb{N}}I$.

The coproduct of CL-objects L_i can abstractly be constructed in the following way: Let L be the topological sum of the L_i. Let F be the set of all continuous maps from L into I which are CL-morphisms on each L_i. Then L is canonically embedded in I^F under evaluation. The CL-subobject of I^F generated by the image of L is the coproduct of the L_i. The closure of the image of L in I^F is, what we would like to know.

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This closure of the image of L is a union of continuous lattices (L_u) which are disjoint except for the identity; one continuous lattice for each ultrafilter on J. The breadth of these lattices does not exceed the maximum breadth of the L_i, if this maximum is finite. In particular, if all the L_i are totally ordered, the same holds for the L_u which come into play. The coproduct of the L_i is isomorphic to the lattice of all closed subsets of the union of the L_u, $u \in \beta J$, which meet every L_u in a filter. These filters may be viewed as lower semicontinuous functions.