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SCS 58: The CL-Compactification of a Continuous Poset

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REFERENCE Rudolf-E.Hoffmann, The CL-compactification of a continuous poset, manuscript (to be circulated, I hope soon, via the informal Proceedings of the second workshop on continuous lattices and continuous posets, Bremen, May 8-1o,1981)

We give several applications (a),(b),(c),(d) of the following result obtained in an earlier paper: The continuous posets P equipped with their Scott topology σ_P are precisely those sober spaces X which have an injective hull λX, i.e. the maximal essential extension λX of X is an injective T_0-space (=continuous lattice in its Scott topology). (a) The CL-topology (=Lawson topology) on P is the trace of the CL-topology of λX, hence it is completely regular Hausdorff. - (b) The CL-closure C of P in λX, the "CL-compactification" of P, is a continuous poset in its own right and the topology induced from the CL-topology of λX is the intrinsic CL-topology. - (c) For a continuous $1,\wedge$-semilattice S, the CL-compactification "coincides" with the injective hull $f:S \to L$, induced by the Scott topology on S, and $f:S \to L$ is characterized by the following properties: L is a continuous lattice and $f:S \to L$ is both a join-dense order-embedding and a dense (topological) embedding with regard to the CL-topologies of S and L, respectively. - (d) The CL-compactification of a continuous poset P "coincides" with the Fell compactification of (P,σ_P). - Thus a construction, the maximal essential extension (in the category T_0) which lives in a non-Hausdorff world bears its fruits in the realm of completely regular and compact Hausdorff spaces. - The CL-compactification of an algebraic poset is an algebraic poset, whereas the class of posets with a.c.c. and the class of partially co-well-ordered sets (a.c.c. and no infinite antichain) are not stable under the CL-compactification (providing, incidentally, continuous posets which are, in their CL-topology, not locally compact and not normal, respectively).

Anyone interested in a copy of the manuscript, before the above mentioned Proceedings are distributed, may write to me.

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