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Demonstration of dynamic thermal compensation for parametric instability suppression in Advanced LIGO

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Abstract

Advanced LIGO and other ground-based interferometric gravitational-wave detectors use high laser power to minimize shot noise and suspended optics to reduce seismic noise coupling. This can result in an opto-mechanical coupling which can become unstable and saturate the interferometer control systems. The severity of these parametric instabilities scales with circulating laser power and first hindered LIGO operations in 2014. Static thermal tuning and active electrostatic damping have previously been used to control parametric instabilities at lower powers but are insufficient as power is increased. Here we report the first demonstration of dynamic thermal compensation to avoid parametric instability in an Advanced LIGO detector. Annular ring heaters that compensate central heating are used to tune the optical mode away from multiple problematic mirror resonance frequencies. We develop a single-cavity approximation model to simulate the optical beat note frequency during the central heating and ring heating transient. An experiment of dynamic ring heater tuning at the LIGO Livingston detector was carried out at 170 kW circulating power and, in agreement with our model, the third order optical beat note is controlled to avoid instability of the 15 and 15.5 kHz mechanical modes. We project that dynamic thermal compensation with ring heater input conditioning can be used in parallel with acoustic mode dampers to control the optical mode transient and avoid

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parametric instability of these modes up to Advanced LIGO's design circulating power of 750 kW. The experiment also demonstrates the use of three mode interaction monitoring as a sensor of the cavity geometry, used to maintain the g-factor product to $g_1 g_2 = 0.829 \pm 0.004$.

Keywords: parametric instability, LIGO, gravitational wave detectors

(Some figures may appear in colour only in the online journal)

1. Introduction

The Advanced LIGO detectors [1] are modified Michelson interferometers that use 4 km long orthogonal arms to measure changes in strain induced by passing gravitational waves. The arms are resonantly enhanced by Fabry–Perot cavities to effectively increase the path length and strengthen the gravitational wave strain signal. Two recycling cavities further enhance the signal: a mirror before the Michelson increases the circulating power and a mirror after the Michelson broadens the bandwidth of the detector. High circulating power is used to decrease shot noise, as this is the limiting fundamental noise source at high frequencies.

Opto-mechanical parametric instability (PI) was first observed in the LIGO detectors in November 2014 when the circulating arm power exceeded 25 kW [2]. These instabilities involve three modes that become coupled in the interferometer: the fundamental optical mode, a higher-order transverse optical mode, and a mechanical resonance of an optic [3], in this case a LIGO test mass. The resulting feedback mechanism can become unstable when radiation pressure from the optical field drives the runaway excitation of the mechanical mode. The severity of instability grows linearly with increased circulating arm power and, if not controlled, can render the detector inoperable.

During the first two observing runs (O1: 9/15–1/16, O2: 9/16–8/17), PIs were controlled using a combination of static thermal compensation and electrostatic damping [2, 4], allowing the power circulating in the arm cavities to be increased up to 120 kW. Annular heaters surrounding the barrels of the test masses are used to alter the radius of curvature (RoC) via thermal expansion, resulting in a static shift of the optical modes away from three mode coincidence. Active electrostatic damping applies forces to the test masses at their mechanical mode frequencies, effectively lowering the Q-factor and reducing the parametric gain to stable levels.

While detector sensitivity during O1 and O2 was sufficient to detect gravitational waves from binary black holes and binary neutron stars [5], higher power is required to reach the Advanced LIGO design sensitivity. As circulating power is increased beyond 120 kW, previous PI control techniques have become insufficient. Static thermal compensation is inadequate because stronger thermal distortions of test mass surfaces result in transverse modes sweeping through a wider frequency band over several hours. Here, we show that this frequency band is larger than the frequency difference between unstable mechanical resonances of the test mass, resulting in instability during parts of this thermal transient. In addition, at higher powers more mechanical modes become near-unstable—many within a few Hz of each other—which further complicates the complex electrostatic damping software architecture as each control signal must be uniquely identified and processed. The mechanical modes are temperature dependent and each test mass experiences a distinct temperature variation; this has led to mode crossing in the error signal which breaks the active damping control loops.

We present the first demonstration of PI avoidance using dynamic thermal compensation (DTC), which was implemented at LIGO Livingston to suppress the transverse mode spacing transient, allowing stable operation at 170 kW intra-cavity power. DTC has previously been demonstrated in an 80 m prototype interferometer [6]: a CO_2 laser was used to replace the heat absorbed from the circulating beam when the interferometer lost lock, thus minimizing the change in cavity geometry. A model of this system was investigated for LIGO [7], however such a system would require extensive additional hardware. We demonstrate a system which expands on static thermal compensation by using the existing ring heaters to dynamically control the RoC of the end test masses and thus maintain the cavity optical mode spacing, offering a fundamental approach to PI control. We also show how measurement of the parametric gain was used to tune DTC, an implementation of three mode interaction monitoring [8].

2. Transient parametric instability in Advanced LIGO

2.1. Parametric instability

PI is a runaway mechanism that can occur when the frequency of an internal mechanical mode of an optic is close to the beat frequency between the fundamental optical mode and a higher-order optical mode present in the cavity. Ambient mechanical motion of the optic's eigenmode scatters light out of the fundamental mode and into a higher-order mode in the cavity. The beat between the two optical modes in turn applies radiation pressure back onto the test mass at the mechanical mode frequency. If the transverse amplitude distribution of this field overlaps with the spatial surface profile of the generating mechanical mode, and if the field is in phase with the motion, the radiation pressure reinforces the surface motion, creating a feedback loop. If the optical energy imparted to the mechanical mode exceeds its dissipation, this results in an instability which can grow beyond the limits of the cavity control systems.

If we model this mechanism as a classic feedback loop [9], the gain of the loop for a particular mechanical mode m in a single cavity is

$$R_m = \frac{8\pi Q_m P}{M\omega_m^2 c\lambda_0}\sum_{n=1}^{\infty}\Re[G_n]B_{m,n}^2 \tag{1}$$

where Q_m is the quality factor of the mechanical mode, P is the circulating power in the cavity, λ_0 is the wavelength of the incident optical field, M is the mass of the optic, ω_m is the mechanical mode frequency, $\Re[G_n]$ is the real part of the complex optical transfer function between the mechanical mode and optical field within the cavity, and $B_{m,n}$ quantifies the spatial overlap between the optical field pressure distribution and the mechanical motion pattern. The summation is over all higher-order optical modes which contribute to R_m. The linear dependence on power makes parametric instability a greater threat as circulating power is increased.

2.2. Transient parametric instability

The LIGO test masses have nominal coating absorption of 0.3–0.5 ppm. When the cavity becomes locked (controlled to remain resonant) and power circulating in the arms increases to hundreds of kW, the absorbed power increases to hundreds of mW in the central portion of the test mass. The subsequent radial temperature gradient from this central heating results in a time-dependent surface deformation of the high-reflectively side of the test mass, which can be approximated as an increase in the RoC at a rate of 105 mW^{-1} of absorbed power [10].

For this analysis, we use a single-cavity model to approximate the optical mode behavior in a LIGO arm cavity (see section 5 for the limitations of this model). In a cavity of length L, the optical beat note spacing $\Delta\omega_{pq} = |\omega_{00} - \omega_{pq}|$ between the Hermite–Gaussian modes

HG_{00} and HG_{pq} modes is dependent on RoC such that:

$$\Delta\omega_{pq} = (p+q)\frac{c}{L}\cos^{-1}\sqrt{\left(1-\frac{L}{r_1}\right)\left(1-\frac{L}{r_2}\right)} \tag{2}$$

where $r_{1,2}$ are the radii of curvature of each optic. In the single-cavity configuration, the optical transfer function is

$$\Re[G_{pq}] = \frac{c}{L\pi\gamma(1+\Delta\omega^2/\gamma^2)} \tag{3}$$

where γ is the optical mode linewidth, and $\Delta\omega = \Delta\omega_{pq} - \omega_m$ describes the frequency spacing between the optical beat note and the mechanical mode. Thus, the coincidence of the three mode interaction is modified by a change in the RoC of a test mass, which alters G_n in equation (1). As the mirrors go through a central heating thermal transient, their increasing RoC sweep the cavity optical mode up in frequency by hundreds of hertz, changing how the optical beat note frequency overlaps with the frequencies of the mechanical modes. The temperature dependent mechanical mode shifts are only a few Hz under typical conditions and thus have negligible impact on transient parametric gain.

The third order optical beat note, $\Delta\omega_{03}$, of the LIGO cold arm cavity sits near 15 kHz. There are test mass mechanical resonances at 15kHz and 15.5kHz that have been observed to cause PI starting at 50kW circulating power [4]. Instability can be prevented if $\Delta\omega_{03}$ avoids these two mechanical mode frequency bands. During power up to 170 kW circulating power, $\Delta\omega_{03}$ shifts up in frequency by $\sim$ 700Hz, sweeping through both of these mechanical mode groups. The thermal transient is slow enough that parametric instability can occur during these times of frequency overlap and cause the interferometer to lose lock.

Annular radiative heating elements encircling the barrels of the LIGO test masses near the rear (anti-reflective) face compensate central heating. The temperature increase causes differential thermal expansion. The net result of the stress field from the larger expansion at the rear of the optic is a quadratic surface change on the front surface equivalent to a decreasing ROC. From equation (2), a decreasing RoC results in a decrease in $\Delta\omega_{pq}$ as L/r_n approaches 2. At the time of our measurements, LIGO Livingston was operating with static ring heater powers that placed $\Delta\omega_{03}$ at just below 14.8 kHz in the absence of central heating.

During O1 and O2, the LIGO Livingston detector was operating with up to 100 kW circulating power. Static thermal tuning was sufficient to avoid parametric gain greater than unity. The central heating transient, offset by the static ring heater setting, resulted in an increase in RoC and an associated increase in $\Delta\omega_{03}$ of a few hundred hertz to a steady state around 15.3 kHz. At this circulating power, the parametric gain was low enough that the amplitude transient of the 15 kHz modes was small.

The sensitivity of the detector is fundamentally limited by shot noise at high frequencies, which depends on the power circulating in the interferometer. To reach Advanced LIGO design sensitivity, higher circulating power is required to reduce the shot noise. At the time of our experiment, the power had been increased to 170 kW circulating in the arm cavities. The increased power absorbed in the central region of a test mass increased the RoC by approximately 10 m and increased the steady state $\Delta\omega_{03}$ by several hundred Hz.

An illustration of the opto-mechanical interaction at 170 kW circulating power (corresponding to 40 W input power) is shown in figure 1. The cyan curve shows the third order optical beat note peak in the cold arm cavity; as power is absorbed in the test mass, this beat note sweeps up in frequency. Mechanical modes are shown in blue. Mode groups A and E are potentially unstable at 170 kW and the red curves show the locations of the third order optical beat note

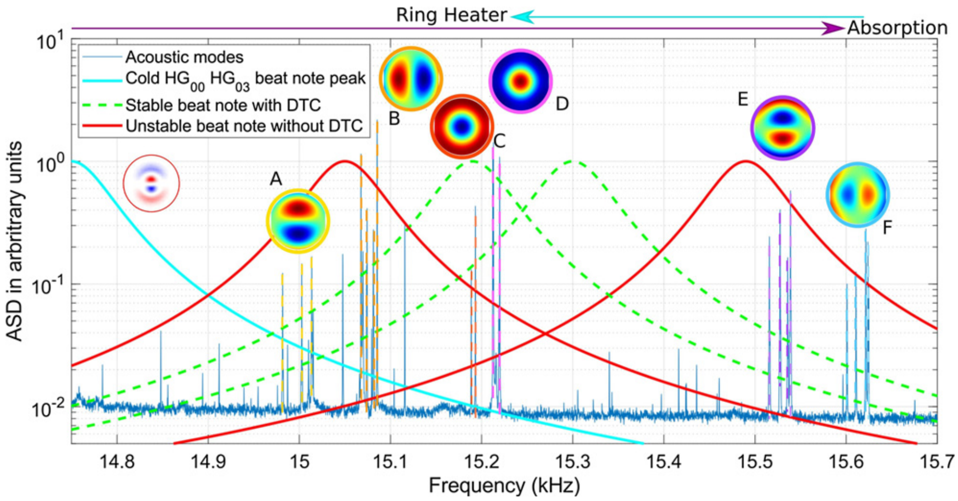

Figure 1. Representation of the mechanical modes (blue trace, one peak per test mass within each mode group) and optical beat note responsible for parametric instability during DTC experiments. Mechanical modes are grouped A–F by mode shape; test mass surface deformation associated with each mode is overlaid. Each surface map is outlined in a colour corresponding to vertical dashed colour lines marking the four associated mechanical modes. The cyan optical beat note shows $\Re[G_{pq}]$ as a function of $\Delta\omega$ for the cold arm cavity; the simulated optical mode is overlaid. Central heating sweeps the optical mode up in frequency. The red curves show locations of instability during this thermal transient. The green curves show the stable tuning range maintained with DTC after the first hour of power up.

at which instability occurred during the central heating thermal transient. Mode groups C and D are stable, as the optical modes that can be excited from these mechanical modes—second and fourth order—are far from resonance. Mode groups B and F are also stable; measurements of mode quality factors Q_m [11] indicate this is primarily due to lower optical gain G_{30} of the HG_{30} mode.

In an attempt to avoid the 15.5 kHz modes, the static ring heater power was increased, but the higher parametric gain with increased circulating power resulted in the 15 kHz modes then becoming unstable for a period. There was no stable static ring heater setting. With more ring heater power, the 15 kHz modes became unstable at the beginning of lock, unlocking the interferometer. Even larger static ring heater settings that pushed the third order optical beat note below 15 kHz resulted in instability of 47.5 kHz mechanical modes due to increased overlap with the $HG_{00}HG_{02}$ optical beat note. With less ring heater power, the 15.5 kHz modes became unstable, unlocking the interferometer about an hour into the lock; an example of this case is shown in figure 2(a).

3. Dynamic thermal compensation

As increased operating power causes the optical mode transient to sweep through larger frequency ranges, a static ring heater setting becomes insufficient for avoiding parametric instability. Dynamic thermal compensation (DTC) is needed to control the optical mode to within a frequency band of sub-unity parametric gain. We demonstrate a technique of applying power to the end test mass ring heaters in multiple steps throughout the transient to compensate the

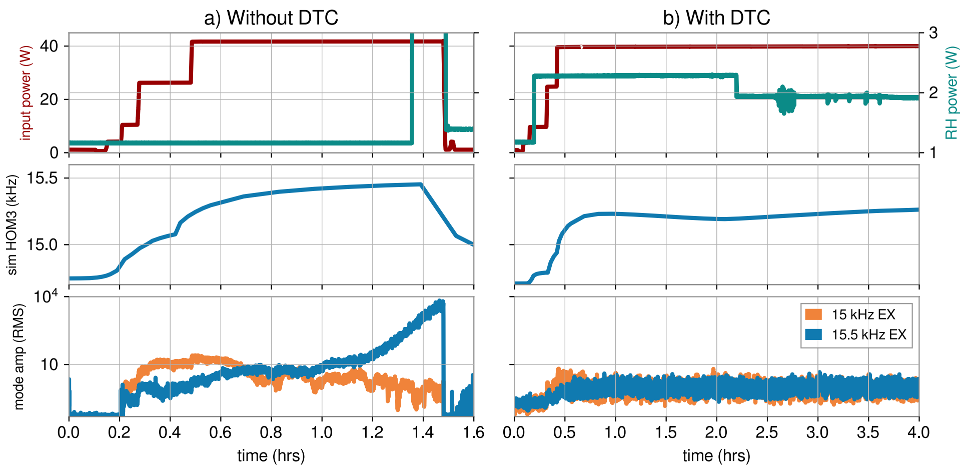

Figure 2. Simulation and experimental results from static and dynamic thermal compensation at LIGO Livingston. The top panels show input laser power and ring heater (RH) power during lock acquisition and into the lock, the middle panels show the single-cavity simulation of the HOM3 spacing, and the bottom panels show mechanical modes' RMS amplitude. (a) Shows the results of a lock at 40 W input power without DTC: as the power is increased, the HOM3 spacing increases. As the optical beat note moves towards 15.5 kHz, the parametric gain of a 15.5 kHz mechanical mode grows and becomes unstable. The ring heater power was stepped up in a last attempt to save the lock, but the resultant optical shift was too slow and PI caused the interferometer to lose resonance. (b) Demonstrates a successful 40 W lock using DTC: the end ring heaters are stepped up prior to input power-up to compensate the initial self heating transient. Two hours later, the ring heaters are stepped down to account for the different thermal transient shapes. Simulation shows the optical mode remaining in the safe zone between 15 kHz and 15.5 kHz and, indeed, mechanical modes remain stable throughout the duration of the lock.

central heating from the cavity beam while accounting for the different shapes of the ring heater and central heating RoC transients. DTC minimizes the change in cavity geometry; in the experiment presented here, $\Delta\omega_{03}$ is controlled to avoid the 15 and 15.5 kHz mechanical modes known to go unstable.

The ring heater and central heating have different RoC step responses, shown in figure 3. The RoC behavior due to central heating is asymptotic, reaching its approximate steady-state value within the first hour. The ring heater transient has an initial overshoot as the heat first hits the outer edge, reaching the maximum RoC decrease around 2 hours, followed by a long, slightly increasing RoC transient as the heat is distributed throughout the test mass until steady state is reached about 12 hours after being changed.

The ring heater has a 15 min delay before a RoC response is observed; therefore it is changed 15 min before the interferometer input power is increased. It is stepped up to a power higher than required for steady state, to better compensate the faster self-heating transient during the beginning of the lock. After about 2 hours, the ring heater power is reduced to steady-state settings; the steady-state end test mass ring heater power is set to compensate an absorption of 0.5 ppm per test-mass.

A simulation was made of the optical mode frequency using the methodology described by Hamedan *et al* [7]. Using finite element modeling software (COMSOL [12]), the time dependence of the surface deformation of each test mass was obtained by applying thermal loads.

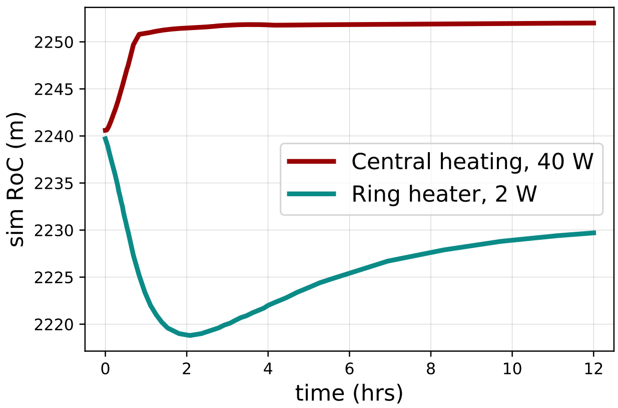

Figure 3. Simulation of end test mass RoC transients due to 2 W ring heater power and central heating from 40 W input power, respectively. Input power has been scaled to approximate 170 kW circulating arm power.

Central heat loads were applied to both input and end test masses and annular heat loads simulating the ring heaters were applied only to the end test masses. The surface deformation was then fit to estimate the RoC transient of each test mass. The time-dependent optical beat note transient for the simple cavity case was calculated using this RoC transient and equation (2), with $(m + n) = 3$. As in [7], this technique was verified with OSCAR (a Matlab based optical FFT code) [13] using the complete mirror maps. For this analysis, we consider only the transient optical beat note frequency between the fundamental mode and the third higher-order mode (HG_{03}) which is responsible for the most problematic PI observed in Advanced LIGO.

Simulations with variable ring heater settings were used to model the optimal DTC necessary for 170 kW. Two simulated cases are shown in the middle panels of figure 2: a 40 W input power (170 kW circulating power) lock without DTC and the same power lock with successful DTC. Without DTC, the static ring heater power used during O2 (1.2 W) was applied to the end test mass, along with a central heating thermal load. For DTC, the ring heater power was increased from 1.2 W to 2.4 W 15 min before applying the central heating, followed by a decrease to 2 W after 2 hours. As 1.2 W ring heater power led to stability at 100 kW circulating power, an additional but moderate dynamic correction was needed at 170 kW to maintain the 100 kW configuration.

Simulation parameters are summarized in table 1. Absorption values for each test mass were extracted from direct measurement of power dependent wavefront distortion with Hartmann wavefront sensors [14]. Accuracy of the ring heater simulation was checked against measurements taken at LIGO, where the ring heater power was stepped up and cavity scans using an auxiliary laser were performed to verify transient mode spacing behavior [15].

Experimental verification of the modeled DTC scheme was carried out in the LIGO Livingston detector and was successfully used to avoid PI with 170 kW circulating power, as shown in figure 2(b). The green curves in figure 1 show the range of the optical mode maintained by DTC after the first hour of lock. DTC was successfully demonstrated over multiple 40 W locks up to 14 hours long.

The Advanced LIGO design calls for 750 kW circulating power in the arms. Assuming 0.5 ppm absorption per test mass, compensating the central heating transient to hold $\Delta\omega_{03}$ stable around 15.3 kHz would require approximately 16 W ring heater power per arm. The ring heaters can deliver up to 40.5 W per test mass [15] and compensation can be distributed

Table 1. The specification of Advanced LIGO LLO X-arm optics used for the single-cavity simulation. Cavity length, input and end test mass (ITM, ETM) parameters, and ring heater (RH) powers are listed.

Features	LLO (X-arm)
Cavity length (m)	4000
ITM RoC (m)	1937.9
ETM RoC (m)	2239.7
ITM-thickness × diameter (m)	0.20 × 0.34
ETM-thickness × diameter (m)	0.199 × 0.34
ITM coating absorption (ppm)	0.3
ETM coating absorption (ppm)	0.5
Beam spot size on ITM (m)	0.055
Beam spot size on ETM (m)	0.062
ITM RH power (W)	0
ETM RH power (W)	Variable

between both the input and end test masses. However, as the power is increased, the range of RoC over which 15 kHz and 15.5 kHz will be parametrically unstable will widen such that no stable RoC region will exist between the two mechanical mode frequency bands. Thus this DTC approach will require mechanical mode Q-factor reduction from passive dampers (see section 6.1) that is roughly equal to the power increase factor.

4. Three mode interaction monitoring

Determination of real-time optical mode frequency spacing currently relies on simulation. Hartmann wavefront sensors [14] monitor changes in wavefront distortion of a beam transmitted through the optic. The RoC of an optic can be inferred from the wavefront distortion by assuming a thermal model; typically such models assume that laser power is absorbed uniformly in the mirror coating. Auxiliary laser systems can be also used to measure the transverse mode spacing of a cavity when the detector is not operating [15]. These techniques have been used to estimate rates of geometry change, such as to calibrate the effect of the ring heater. However, the cavity geometry during high power operation is affected by other things, such as thermal expansion from nonuniform absorption in the coatings [16], and by the beam position on the test mass [17] in conjunction with figure error in the mirror surface.

Each mechanical mode signal can be measured on photodetectors in transmission of the arm cavity or at the dark port of the interferometer as a beat between the main circulating light and the light scattered from the mechanical mode. The signal is the result of spatial overlap between the mechanical mode and one or more transverse optical cavity modes. The signal is enhanced if the scattered light is resonant in the cavity, i.e. maximising equation (3). When the opto-mechanical coupling is high enough, the interaction results in a change in mechanical mode physical amplitude (as opposed to only signal amplitude). As these interactions depend strongly on the optical beat note frequency, they act as a highly sensitive witness of cavity geometry. Monitoring the mechanical mode amplitude has therefore been suggested as a tool to monitor cavity geometry [8]. The experiment reported in this paper provides a demonstration of a three mode interaction monitoring technique for the full operating LIGO interferometer. In this case, limits are inferred on the allowed cavity geometry consistent with a parametrically stable model.

Parametric instability was used to tune the DTC. This was done by using the observation of instabilities to inform the choice of heater settings. Minor changes from the tuned heater setting resulted in instability such as that shown in figure 2 (15.5 kHz) or in the 15 kHz mode. This tuning therefore demonstrated the cavity geometry was maintained between the two unstable thermal tuning regimes for 15 kHz and 15.5 kHz mechanical resonances. Assuming the simplest model of a cavity with an unknown line-width we can estimate the cavity g-factor from the knowledge that the optical beat-note must lie between 15 kHz and 15.5 kHz. Using equation (2), we estimate the cavity g-factor product has been maintained to within $g_1g_2 = 0.829 \pm 0.006$ after an initial thermal transient of approximately one hour, where $g_\mathrm{n} = 1 - (L/R_n)$. For a complete interferometer with multiple coupled cavities, the simulated stable range is considerably more narrow. In figure 1, the parametric gain as a function of mirror RoC are compared for a single cavity and the full interferometer. Where orange and red traces are below one a simple cavity is stable while where green and blue are below 1 the full interferometer simulation is stable. The estimated g-factor range assumes a single-cavity model approximation, a full interferometer model reduces the g-factor range between unstable regimes. Therefore, this estimate provides an upper limit on g-factor stability range.

In addition to avoiding PI, maintaining the cavity geometry reduces the change in cavity beam parameters and stabilizes mode matching to the input and output of the interferometer. This will be necessary to achieve the ambitious loss requirements of Advanced LIGO and A+. In principle, a much higher precision g-factor measurement could be made from the optomechanical interaction strength; however, this is conditional on having a good understanding of the resonant optical modes discussed in section 5 and a reliable measurement of the interaction strength.

5. Optical model extension

The model used to tune the DTC approximates the optical system as a single cavity rather than using Advanced LIGO's full core configuration: a dual-recycled Michelson interferometer with Fabry–Perot arms (DRFPMi). This simplifies the analysis by allowing direct calculation of the time-dependent optical beat-note spacing of a single arm cavity via equation (2). The approximation is efficient for simulating general opto-mechanical overlap behavior where a precision of $\sim$100 Hz is sufficient. As the unstable mechanical mode density increases, and for the higher precision application of 3MIM as a cavity control mechanism, a more complete model of parametric gain within the full coupled cavity system must be utilized.

It has been shown that the presence of the power- and signal-recycling cavities alter the resonant conditions for higher order modes in the interferometer and therefore the conditions for which a mechanical mode will produce a PI [18, 19]. When the full interferometer configuration is considered, the parametric gain response with respect to changes in RoC alters from a single broad peak to multiple narrower, taller peaks, as shown in figure 4.

Figure 4 is the result of a Finesse model [20] configured as described by Green *et al* [19], with input power re-scaled to give an arm circulating power of 170 kW as was used in the experiment. In each case, a surface motion map, generated using a very simple Comsol model of the optic [21], is applied to ETMX at its eigenfrequency. The curvature of both ETMs in the Finesse model are then scanned simultaneously and the resulting parametric gain computed. This is done for both the full DRFPMi model and a reduced version which excludes all cavities except the X-arm, with input power re-scaled to compensate. Note that the mechanical mode amplitude distribution at the surface is generated for a flat optic, with all curvature introduced directly in the Finesse model.

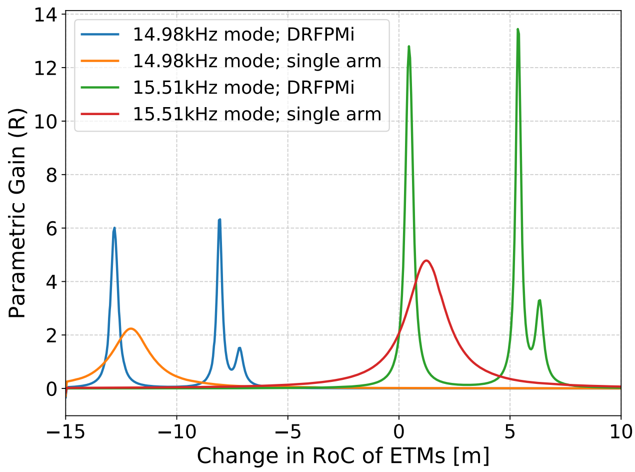

Figure 4. The parametric gain of the 15 kHz and 15.5 kHz mechanical modes as a function of ETMX and ETMY common radius of curvature change (from 2248 m). When the full DRFPMi is taken into account, the range of RoCs for which the interferometer is stable for both modes is split and reduced in width compared to the single arm model. In both configurations, input power is scaled to give 170 kW arm circulating power, and a Q-factor of 1×10^7 is used for both mechanical modes (in agreement with measured values to within a factor of three).

When the recycling cavities are included, the range of RoC curvatures for which each of the 15 kHz and 15.5 kHz modes are stable becomes split, and spread over a wider range. While each individual unstable region is narrower than suggested by the single arm model, the stable regions are also expected to be narrower: in the single arm model, the stable region between the two modes is approximately 10 m, while for the full interferometer this is reduced to approximately 7 m.

This model represents a single snapshot in time of the interferometer state; in practice, the properties of the recycling cavities vary slightly from lock to lock and during the thermal transient. This could smear the effect of the recycling cavities when measuring R vs RoC experimentally.

6. Future applications

Since these experiments were performed, there have been several upgrades to the LIGO detectors. Passive dampers have been installed on the test masses to reduce the Q-factors of the test mass mechanical modes and decrease the severity of PI. The ring heater response has been filtered such that a more precise analogue of the central heating response can be produced with the ring heater. Additionally, the power has been increased to approximately 230 kW in the arm cavities.

6.1. Acoustic mode dampers

Thermal tuning controls PI by changing the optical gain G_n in equation (1). The parametric gain of a particular mode can also be suppressed by reducing the Q-factor of the mechanical mode involved in the three mode interaction. Prior to O3, acoustic mode dampers were installed on all test masses in LIGO to passively reduce the Q-factors of a large number of mechanical

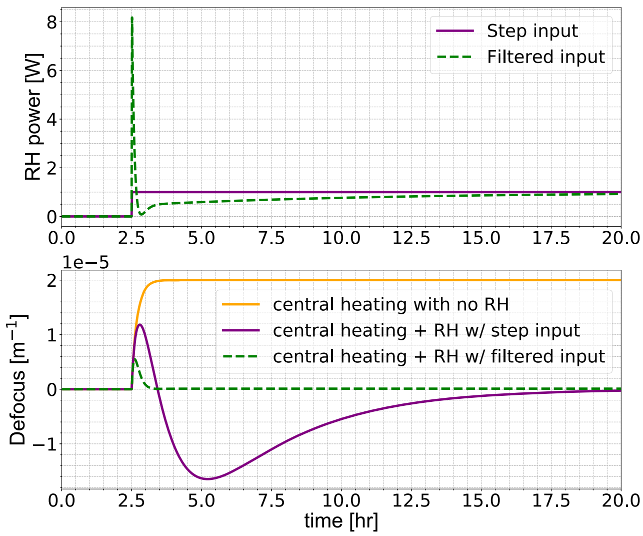

Figure 5. Comparison between the natural transient response of the ring heater (RH) vs the response with ring heater input filtering. The central heating data used here was generated by a COMSOL model of the test mass with 1 W of central absorbed power (with a resultant lensing of 947 uD/W_{abs} scaled to 21 mW absorbed power. For the measured ITM absorption values in O3, we estimate that the above curve would be produced from 100 kW of circulating arm power.

modes [11]. With the exception of one 15 kHz mode, Q-factors were reduced by approximately an order of magnitude or more in the 15–80 kHz frequency range, nearly eliminating PI for Advanced LIGO. Several modes will likely still become unstable at design circulating power (750 kW), including a 15.5 kHz mode. However, as acoustic mode dampers have reduced the Q-factors for the 15 kHz modes by at least the same factor as the designed power increase, the stable region is expected to remain at least as broad as discussed in section 5. DTC could be used to maintain the cavity optical mode spacing to within this region.

Since acoustic mode damper installation, modes at 10.2 kHz and 10.4 kHz have been observed to become unstable at LIGO Hanford with 230 kW circulating power. This is thought to be due to increased noise coupling and changes in spatial overlap ($B_{m,n}$ in equation (1)) due to intentional beam de-centering [11]. At 230 kW, these instabilities have been avoided by using static thermal compensation to shift the second order optical modes by $\sim$100 Hz. At design power, it is projected that 10 kHz and 15 kHz modes may both become unstable even with acoustic mode dampers. Using DTC for PI avoidance across multiple optical-mechanical mode group overlaps requires a global stable operating point: as DTC shifts all optical modes, it is possible that decreasing the opto-mechanical frequency overlap for one mechanical mode group will increase the overlap for another. If thermal tuning is used to maintain $\Delta\omega_{03} \sim$ 15.3 kHz, then the second order optical beat note will be approximately 10.1 kHz. A complete understanding of the unstable mode shapes, their parametric gain, and the effect of beam decentering will be needed to determine if DTC will be sufficient to simultaneously avoid multiple instabilities.

6.2. Ring heater input conditioning

Thermal compensation for cavity control can be improved if the lens response from the ring heater more exactly offsets the central heating response. As shown in figure 3 and discussed in section 3, the central heating transient reaches steady state after approximately an hour, while the ring heater results in an overshoot of the lens response followed by a transient with a time constant over 12 hours. It is possible to reduce this time constant and eliminate the overshoot with real-time digital filtering of the ring heater step input [22]. While signal conditioning is not a novel method, it is significant improvement to the central heating compensation will make it necessary for DTC as both circulating and ring heater power increases. The filtered ring heater input and resultant thermal lens in congruence with central heating is shown in figure 5.

The ring heater conditioning technique was successfully tested at both detectors. The interferometer was locked and Hartmann wavefront sensors were used to measure the substrate thermal deformation transient due to central heating [14]. Once the system had thermally stabilized, the ring heaters were stepped up by 0.4 W with the conditioning filters engaged and the resultant lensing was measured in the same way. The measured transient time constant of the ring heater differed by only 15 min from that of the central heating, indicating reasonable transient compensation. This technique is limited by available ring heater power during the large initial transient. As compensation for 100 kW circulating power required 8 W ring heater power and a single ring heater delivers up to approximately 40 W, we estimate that ring heater conditioning can fully compensate up to 500 kW circulating power per test mass.

The usefulness of shaping the ring heater transient extends beyond PI suppression. Filters have also been tested that provide fast transients ($\sim$1000 s) for small steps, allowing near instantaneous control of the cavity g-factor. This fast feedback time will be helpful for real-time cavity geometry control as discussed in section 4.

7. Conclusions

We have described the implementation of a DTC scheme using existing ring heaters to avoid parametric instability in Advanced LIGO when operating at 170 kW circulating cavity power. We use a simplified single-cavity model to predict the behavior of PI during the thermal transient from cold to full-power state. This model was verified through experimentation, correctly approximating the thermal transient to within 100 Hz. We project that this DTC scheme can be used to avoid parametric instability in the 15 and 15.5 kHz modes at the full Advanced LIGO circulating power with acoustic mode dampers installed. A more complete characterization of additional unstable mechanical and optical modes will be needed to determine if DTC can be used to avoid instability from multiple optical modes simultaneously.

We demonstrate experimentally how monitoring the opto-mechanical three mode interaction can be used to maintain the cavity g-factor between $0.825 < g_1 g_2 < 0.833$. Three mode interaction monitoring may be used to validate more complete parametric instability models that include the full interferometer configuration; such a model will be necessary as increased power tightens the bounds on stable frequency regimes.

DTC will be improved by pre-filtering the ring heater to produce a better approximation of the self-heating transient. DTC may also be improved by adding input test mass compensation, enabling more complete mode healing. Additionally, it would reduce thermal transients in the input test mass thermal lens, stabilizing beam size at the interferometer output and transient contrast defects.

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