Journal of Stochastic Analysis

Volume 1 | Number 1 Article 5

March 2020

Exit Problems for Jump-Diffusion Processes with Uniform Jumps

Mario Lefebvre
Polytechnique Montréal, Montréal, Québec H3C 3A7, Canada, mlefebvre@polymtl.ca

Journal of Stochastic Analysis
Vol. 1, No. 1 (2020) Article 5 (9 pages)
DOI: 10.31390/josa.1.1.05

LSU Libraries
digitalcommons.lsu.edu/josa

EXIT PROBLEMS FOR JUMP-DIFFUSION PROCESSES WITH UNIFORM JUMPS

MARIO LEFEBVRE*

Abstract. The problem of computing the moment-generating function of the first exit time $T(x)$ from the interval (a,b) for a time-homogeneous jump-diffusion process $X(t)$, starting from $X(0)=x$, is considered. The jump sizes are assumed to be uniformly distributed. Exact results are obtained when the jumps can be large, as well as approximate analytical solutions when the jumps are small. The mean of $T(x)$ and the probability $P[X(T(x)) \le a]$ are also computed in important cases.

1. Introduction

Let $\{B(t), t \ge 0\}$ be a standard Brownian motion and $\{N_i(t), t \ge 0\}$ be Poisson processes, with rate λ_i, for $i=1,2$. The three stochastic processes are assumed to be independent. Moreover, let $\{Y_i, i=1,2,\ldots\}$ be independent and identically distributed (i.i.d.) random variables having the common probability density function

$$f_Y(y) = \frac{1}{\beta_1} \quad \text{for } 0 \le y \le \beta_1. \tag{1.1}$$

Similarly, $\{Z_j, j=1,2,\ldots\}$ are i.i.d. random variables having the common probability density function

$$f_Z(z) = \frac{1}{\beta_2} \quad \text{for } -\beta_2 \le z \le 0. \tag{1.2}$$

The random variables Y_i and Z_j are also assumed to be independent between themselves.

We consider a jump-diffusion process $\{X(t), t \ge 0\}$ whose continuous part is a time-homogeneous diffusion process with infinitesimal mean $\mu[X(t)]$ and infinitesimal variance $\sigma^2[X(t)]$. Moreover, there are Poissonian jumps, so that $X(t)$ is such that

$$X(t) = X(0) + \int_0^t \mu[X(s)]\,\mathrm{d}s + \int_0^t \sigma[X(s)]\,\mathrm{d}B(s) + \sum_{i=1}^{N_1(t)} Y_i + \sum_{i=1}^{N_2(t)} Z_i. \tag{1.3}$$

Received 2019-11-17; Accepted 2020-1-28; Communicated by the editors.
2010 *Mathematics Subject Classification.* Primary 60J75; Secondary 60J60.
Key words and phrases. First exit time, Wiener process, Poisson process, jump size, integro-differential equation.
* Corresponding author.

Next, let $T(x)$ be the first exit time of $X(t)$ from the interval (a, b):

$$T(x) = \inf\{t \geq 0 : X(t) \notin (a, b) \mid X(0) = x \in [a, b]\}. \tag{1.4}$$

In this paper, we are interested in computing the moment-generating function of $T(x)$, as well as its mean and the probability $P[X(T(x)) \leq a]$.

Jump-diffusion processes are used in mathematical finance as models for the evolution of stock prices (see Merton [9] and Dong and Han [4]), and also in reliability theory (Ghamlouch *et al.* [5]). First-passage-time problems for these jump-diffusion processes have been considered, among others, by Abundo [1] and Kou and Wang [7] (see also the references therein). Peng and Liu [10] computed the moments of first-passage times for jump-diffusion processes with Markovian switching. Bo and Lefebvre [3] worked on mean first-passage times for two-dimensional diffusion processes with jumps.

In Abundo [1], the author assumed that the jumps (positive and/or negative) were of constant size. Kou and Wang [7] proposed an asymmetric double exponential distribution for the jump sizes, while Dong and Han [4] used a hyper-Erlang distribution. Other possibilities are log-normal (Merton [9]) and log-uniform (Ahlip and Prodan [2]) distributions.

In Section 2, exact results will be obtained for important particular processes. In Section 3, we will present examples that show that it is possible to obtain good approximations to the exact solutions when the jump sizes are small. We will end with a few concluding remarks in Section 4.

2. Exact Results

Assume that $f(x)$ is a twice continuously differentiable function. The infinitesimal generator of the process $\{X(t), t \geq 0\}$ defined in (1.3) is

$$\begin{aligned} \mathcal{L}f(x) &= \frac{1}{2}\sigma^2(x) f''(x) + \mu(x) f'(x) - (\lambda_1 + \lambda_2) f(x) \\ &\quad + \frac{\lambda_1}{\beta_1}\int_0^{\beta_1} f(x+u)\,\mathrm{d}u + \frac{\lambda_2}{\beta_2}\int_{-\beta_2}^{0} f(x+v)\,\mathrm{d}v \end{aligned} \tag{2.1}$$

for $x \in (a, b)$. If we assume further that the conditional transition density function

$$p(x_1, t; x_0, t_0) := \lim_{dx_1 \downarrow 0} \frac{P[X(t) \in (x_1, x_1 + dx_1] \mid X(t_0) = x_0]}{dx_1} \tag{2.2}$$

exists for $t > t_0$ (see Gihman and Skorohod [6]), then we can write that the moment-generating function of the random variable $T(x)$, namely

$$M(x) := E\left[e^{-\alpha T(x)}\right], \tag{2.3}$$

where $\alpha > 0$, satisfies for $x \in (a, b)$ the integro-differential equation

$$\begin{aligned} \alpha M(x) &= \frac{1}{2}\sigma^2(x) M''(x) + \mu(x) M'(x) - (\lambda_1 + \lambda_2) M(x) \\ &\quad + \frac{\lambda_1}{\beta_1}\int_0^{\beta_1} M(x+u)\,\mathrm{d}u + \frac{\lambda_2}{\beta_2}\int_{-\beta_2}^{0} M(x+v)\,\mathrm{d}v. \end{aligned} \tag{2.4}$$

The boundary condition is

$$M(x) = 1 \quad \text{if } x \notin (a,b). \tag{2.5}$$

We can prove the following proposition.

Proposition 2.1. *Suppose that the function $M(x)$ defined in (2.3) is three times differentiable, and that $\sigma^2(x)$ and $\mu(x)$ are differentiable functions. If $a+\beta_1 \geq b$ and $b-\beta_2 \leq a$, then $M(x)$ satisfies the third-order ordinary differential equation (o.d.e.)*

$$\begin{aligned} 0 &= \frac{1}{2}\sigma^2(x)\,M'''(x) + \left(\frac{1}{2}[\sigma^2(x)]' + \mu(x)\right) M''(x) \\ &\quad + [\mu'(x) - \alpha - \lambda_1 - \lambda_2]\; M'(x) + \left(\frac{\lambda_2}{\beta_2} - \frac{\lambda_1}{\beta_1}\right) M(x) - \frac{\lambda_1}{\beta_1} + \frac{\lambda_2}{\beta_2} \end{aligned} \tag{2.6}$$

for $a < x < b$.

Proof. Equation (2.4) can be rewritten as follows:

$$\begin{aligned} \alpha\, M(x) &= \frac{1}{2}\sigma^2(x)\,M''(x) + \mu(x)\,M'(x) - (\lambda_1+\lambda_2)\,M(x) \\ &\quad + \frac{\lambda_1}{\beta_1}\int_x^{x+\beta_1} M(y)\,\mathrm{d}y + \frac{\lambda_2}{\beta_2}\int_{x-\beta_2}^{x} M(z)\,\mathrm{d}z. \end{aligned} \tag{2.7}$$

The result is then obtained by differentiating the above equation and using the fact that $M(x+\beta_1) = M(x-\beta_2) = 1$ for all x in the interval (a,b). □

Remark 2.2. (i) The general solution of Eq. (2.6) will contain three arbitrary constants. When a and b are both finite, we can write that

$$M(a) = M(b) = 1. \tag{2.8}$$

Moreover, by definition, $0 < M(x) \leq 1$. This condition is useful when $a = -\infty$ or $b = \infty$; that is, in the case of a one-barrier problem.
(ii) If we can determine the values of two constants, we can then substitute the expression obtained into Eq. (2.4) to find the remaining constant. In some cases, symmetry can be used to obtain a third condition.

In the case of the function

$$m(x) := E[T(x)], \tag{2.9}$$

the left-hand side of Eq. (2.4) is replaced by -1, while it is replaced by 0 for

$$p(x) := P[X(T(x)) \leq a]. \tag{2.10}$$

Proceeding as above, we obtain the following corollaries, in which we assume that $\sigma^2(x)$ and $\mu(x)$ are differentiable functions.

Corollary 2.3. *Suppose that the function $m(x)$ defined in Eq. (2.9) is three times differentiable. If $a+\beta_1 \geq b$ and $b-\beta_2 \leq a$, then $m(x)$ satisfies the o.d.e.*

$$\begin{aligned} 0 &= \frac{1}{2}\sigma^2(x)\,m'''(x) + \left(\frac{1}{2}[\sigma^2(x)]' + \mu(x)\right) m''(x) \\ &\quad + [\mu'(x) - \lambda_1 - \lambda_2]\; m'(x) + \left(\frac{\lambda_2}{\beta_2} - \frac{\lambda_1}{\beta_1}\right) m(x) \end{aligned} \tag{2.11}$$

for $a < x < b$, subject to the boundary condition

$$m(x) = 0 \quad \text{if } x \notin (a,b). \tag{2.12}$$

Corollary 2.4. *Suppose that the function $p(x)$ defined in Eq. (2.10) is three times differentiable. If $a + \beta_1 \geq b$ and $b - \beta_2 \leq a$, then we can write that*

$$\begin{aligned} 0 &= \frac{1}{2}\sigma^2(x)\,p'''(x) + \left(\frac{1}{2}[\sigma^2(x)]' + \mu(x)\right) p''(x) \\ &\quad + [\mu'(x) - \lambda_1 - \lambda_2]\,p'(x) + \left(\frac{\lambda_2}{\beta_2} - \frac{\lambda_1}{\beta_1}\right) p(x) - \frac{\lambda_2}{\beta_2} \end{aligned} \tag{2.13}$$

for $a < x < b$, subject to

$$p(x) = 1 \quad \text{if } x \leq a \quad \text{and} \quad p(x) = 0 \quad \text{if } x \geq b. \tag{2.14}$$

2.1. Particular case. Suppose that $\sigma^2(x) \equiv 1$ and $\mu(x) \equiv 0$, so that the continuous part of the process $X(t)$ is a standard Brownian motion, $\lambda_1 = \lambda_2 = 1$, $\beta_1 = \beta_2 = 1$, $a = 0$ and $b = 1$. Equation (2.6) then reduces to

$$M'''(x) - (4 + 2\alpha)\,M'(x) = 0. \tag{2.15}$$

Let

$$\gamma := \sqrt{4 + 2\alpha}. \tag{2.16}$$

The solution of the linear o.d.e. with constant coefficients (2.15) that satisfies the boundary conditions $M(0) = M(1) = 1$ is

$$M(x) = 1 + c_1\,\frac{e^{-\gamma} - e^{\gamma} + (1 - e^{-\gamma})\,e^{\gamma x}}{e^{\gamma} - 1} + c_1\,e^{-\gamma x}, \tag{2.17}$$

where c_1 $(= c_1(\alpha))$ is an arbitrary constant.

By symmetry, we can write that $M'(1/2) = 0$. Unfortunately, this third condition does not enable us to determine the value of the constant c_1. However, if we substitute the function $M(x)$ given in Eq. (2.17) into the equation (see Eq. (2.7))

$$\begin{aligned} \alpha\,M(x) &= \frac{1}{2}M''(x) - 2\,M(x) \\ &\quad + \int_x^1 M(y)\,\mathrm{d}y + \int_1^{x+1} 1\,\mathrm{d}y + \int_{x-1}^0 1\,\mathrm{d}z + \int_0^x M(z)\,\mathrm{d}z, \end{aligned} \tag{2.18}$$

we can deduce an explicit expression for c_1:

$$c_1(\alpha) = \frac{\alpha\,\gamma\,e^{\gamma}}{(\alpha+1)\,\gamma\,(e^{\gamma}+1) + 2\,(e^{\gamma}-1)}. \tag{2.19}$$

The moment-generating function $M_0(x)$ of the random variable $T(x)$ when there are no jumps is easily found to be

$$M_0(x) = \frac{(1 - e^{-\sqrt{2}})\,e^{\sqrt{2}x}}{e^{\sqrt{2}} - e^{-\sqrt{2}}} + \frac{(e^{\sqrt{2}} - 1)\,e^{-\sqrt{2}x}}{e^{\sqrt{2}} - e^{-\sqrt{2}}} \quad \text{for } 0 \leq x \leq 1. \tag{2.20}$$

The functions $M(x)$ and $M_0(x)$ are shown in Figure 1 in the case when $\alpha = 1$. We have $c_1(1) \simeq 0.3426$.

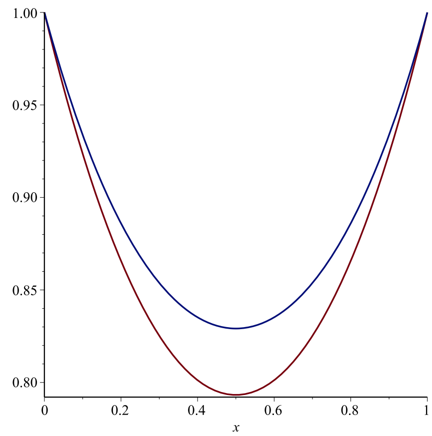

FIGURE 1. Functions $M(x)$ (above) and $M_0(x)$ in the interval $[0, 1]$ when $\alpha = 1$, $\sigma^2(x) \equiv 1$, $\mu(x) \equiv 0$, $\lambda_1 = \lambda_2 = 1$ and $\beta_1 = \beta_2 = 1$.

Next, to obtain $m(x) := E[T(x)]$, we must solve (see Eq. (2.11))

$$\frac{1}{2} m'''(x) - 2m'(x) = 0 \tag{2.21}$$

for $0 < x < 1$, subject to the boundary conditions

$$m(x) = 0 \quad \text{if } x = 0 \text{ or } 1. \tag{2.22}$$

We easily find that

$$m(x) = c_1 \left(e^{2(x-1)} + e^{-2x} - 1 - e^{-2} \right). \tag{2.23}$$

Substituting this function into the integro-differential equation

$$-1 = \frac{1}{2} m''(x) - 2m(x) + \int_x^1 m(y)\,\mathrm{d}y + \int_0^x m(z)\,\mathrm{d}z, \tag{2.24}$$

we conclude that we must choose $c_1 = -1/2$. The functions $m(x)$ and $m_0(x) := x(1-x)$ (when $\lambda_1 = \lambda_2 = 0$) are presented in Figure 2.

Finally, the function $p(x) := P[X(T(x)) \leq 0]$ satisfies the o.d.e. (see Eq. (2.13))

$$\frac{1}{2} p'''(x) - 4p'(x) = 2 \tag{2.25}$$

for $0 < x < 1$. The boundary conditions are

$$p(0) = 1 \quad \text{and} \quad p(1) = 0. \tag{2.26}$$

We have

$$p(x) = c_1 \left(e^{-2x} + \frac{1 - e^{-2}}{e^2 - 1} e^{2x} + \frac{e^{-2} - e^2}{e^2 - 1} \right) - \frac{x}{2} + \frac{2e^2 - 1 - e^{2x}}{2(e^2 - 1)}. \tag{2.27}$$

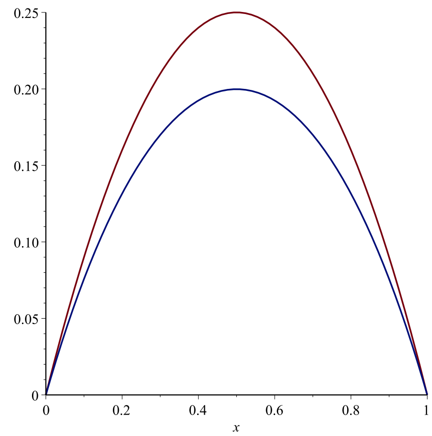

FIGURE 2. Functions $m(x)$ (below) and $m_0(x)$ in the interval $[0,1]$ when $\sigma^2(x) \equiv 1$, $\mu(x) \equiv 0$, $\lambda_1 = \lambda_2 = 1$ and $\beta_1 = \beta_2 = 1$.

We substitute the above function into the equation

$$0 = \frac{1}{2} p''(x) - 2p(x) + \int_x^1 p(y)\,\mathrm{d}y + \int_{x-1}^0 1\,\mathrm{d}z + \int_0^x p(z)\,\mathrm{d}z, \tag{2.28}$$

which enables us to conclude that

$$c_1 = \frac{e^2}{4(e^2 - 1)}. \tag{2.29}$$

When there are no jumps, $p_0(x) := P[X(T(x)) \leq 0] = 1 - x$. The functions $p(x)$ and $p_0(x)$ are almost identical, as can be seen in Figure 3.

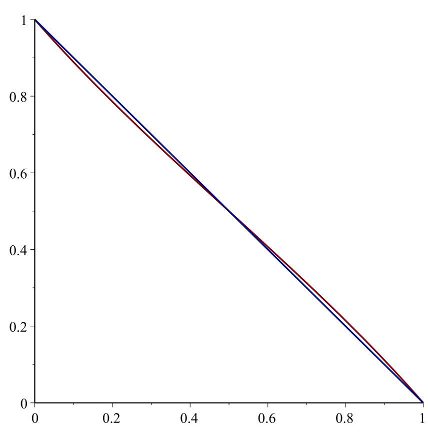

FIGURE 3. Functions $p(x)$ and $p_0(x)$ in the interval $[0,1]$ when $\sigma^2(x) \equiv 1$, $\mu(x) \equiv 0$, $\lambda_1 = \lambda_2 = 1$ and $\beta_1 = \beta_2 = 1$.

In the next section, we will consider the case when β_1 and β_2 are small.

3. Approximate Results for $m(x)$

In the previous section, we obtained exact results which are valid when β_1 and β_2 are large, relative to the length of the interval $[a, b]$. We will now derive approximate results for $m(x)$ when β_1 and β_2 are both small. These approximate results will be exact or almost exact in the subinterval $[a + \beta_2, b - \beta_1]$. We could also try to compute the moment-generating function $M(x)$ and the probability $P[X(T(x)) \leq a]$.

First, when the continuous part of the process $\{X(t), t \geq 0\}$ is a standard Brownian motion and $\lambda_1 = \lambda_2 = 0$, the function $m(x)$ is given by

$$m(x) = -(x-a)\,(x-b) \quad \text{for } a \leq x \leq b. \tag{3.1}$$

Let us look for a solution of the form

$$m(x) = c_1\,(x-a)\,(x-b) \quad \text{for } a \leq x \leq b \tag{3.2}$$

of the integro-differential equation

$$\begin{aligned} -1 \quad = \quad & \frac{1}{2}\sigma^2(x)\,m''(x) + \mu(x)\,m'(x) - (\lambda_1 + \lambda_2)\,m(x) \\ & + \frac{\lambda_1}{\beta_1}\int_x^{x+\beta_1} m(y)\,\mathrm{d}y + \frac{\lambda_2}{\beta_2}\int_{x-\beta_2}^{x} m(z)\,\mathrm{d}z. \end{aligned} \tag{3.3}$$

Suppose that $x \in [a + \beta_2, b - \beta_1]$. If we substitute the function in (3.2) into Eq. (2.11), we find that we must have

$$\begin{aligned} 0 \quad = \quad & [\sigma^2(x)]' + 2\,\mu(x) + (2\,x - a - b)\,[\mu'(x) - \lambda_1 - \lambda_2] \\ & + \lambda_1\,(2\,x + \beta_1 - a - b) + \lambda_2\,(2\,x - \beta_2 - a - b). \end{aligned} \tag{3.4}$$

It follows that $\mu'(x)$ must be equal to zero, which then implies that

$$\mu(x) \equiv \mu_0 := -\frac{1}{2}\left\{[\sigma^2(x)]' + \lambda_1\,\beta_1 - \lambda_2\,\beta_2\right\}. \tag{3.5}$$

We can now state the following proposition.

Proposition 3.1. *If $\mu(x)$ is equal to the constant μ_0 defined in Eq. (3.5), then the function $m(x) := E[T(x)]$ is given by*

$$m(x) = c_1\,(x-a)\,(x-b) \quad \textit{for } a + \beta_2 \leq x \leq b - \beta_1, \tag{3.6}$$

where the constant c_1 is obtained by substituting the above expression for $m(x)$ into the integro-differential equation (3.3).

As a first example, let us take $\sigma^2(x) \equiv 1$, $\lambda_1 = \lambda_2 = 1$, $\beta_1 = \beta_2 = 1/10$, $a = 0$ and $b = 1$. Then we must choose $\mu(x) \equiv 0$. We find that $c_1 \simeq -0.9934$. When we use the function

$$m(x) = -0.9934\,x\,(x-1) \tag{3.7}$$

in the interval $[0.9, 1]$, the difference between the value taken by the right-hand side of Eq. (3.3) and -1 goes from 0 to approximately 0.053 (and, similarly, it goes from 0.053 to 0 in the interval $[0, 0.1]$).

Next, if $\sigma^2(x) = x$ in the interval $[1, 2]$, we must then take $\mu(x) \equiv -1/2$. This time, the constant c_1 is approximately equal to -0.6634. The error in the interval $[1.9, 2]$ increases from 0 to $\simeq 0.035$.

3.1. Special case. To conclude, we will consider the special case when $\lambda_1 = \lambda_2 := \lambda$ and $\beta_1 = \beta_2 := \beta$. Then the integro-differential equation (3.3) can be written as follows:

$$-1 = \frac{1}{2}\sigma^2(x)\, m''(x) + \mu(x)\, m'(x) - 2\lambda\, m(x) + \frac{\lambda}{\beta}\int_{-\beta}^{\beta} m(x+y)\, \mathrm{d}y. \tag{3.8}$$

We have

$$m(x+y) = m(x) + y\, m'(x) + \frac{1}{2}y^2\, m''(x) + \ldots \tag{3.9}$$

Therefore, if β is small, we can write that

$$-1 \simeq \left(\frac{1}{2}\sigma^2(x) + \frac{\lambda\beta^2}{3}\right) m''(x) + \mu(x)\, m'(x). \tag{3.10}$$

As an example, let us assume that $\sigma^2(x) = x^2$ and $\mu(x) = x$, so that $\{X(t), t \geq 0\}$ is a geometric Brownian motion with Poissonian jumps. We take $\lambda = 1$, $\beta = 1/10$, $a = 1$ and $b = 2$. We must then solve the o.d.e.

$$-1 \simeq \left(\frac{1}{2}x^2 + \frac{1}{300}\right) m''(x) + x\, m'(x). \tag{3.11}$$

The solution that satisfies the boundary conditions $m(1) = m(2) = 0$ is

$$\begin{aligned} m(x) &= -\ln(150x^2+1) + \frac{[\ln(601) - \ln(151)]\arctan(5\sqrt{6}x)}{\arctan(10\sqrt{6}) - \arctan(5\sqrt{6})} \\ &\quad + \frac{\arctan(10\sqrt{6})\ln(151) - \arctan(5\sqrt{6})\ln(601)}{\arctan(10\sqrt{6}) - \arctan(5\sqrt{6})} \\ &\simeq -\ln(150x^2+1) + 33.97\arctan(12.25x) - 45.57. \end{aligned} \tag{3.12}$$

With the above function, we find that the difference between the right-hand side of Eq. (3.3) and -1 in the interval $[1.1, 1.9]$ decreases from about -0.000055 to -0.0000031. Hence, the expression obtained for $m(x)$ is almost exact. In the interval $[1.9, 2]$, the error increases to approximately 0.0156. Finally, the error is strictly decreasing in $[1, 1.1]$, with a value of approximately 0.0447 at $x = 1$.

4. Concluding Remarks

In this paper, we obtained explicit solutions to a difficult problem, namely that of computing the moment-generating function and/or the mean of the first exit time $T(x)$ from the interval (a, b) for jump-diffusion processes $\{X(t), t \geq 0\}$, as well as the probability $P[X(T(x)) \leq a]$. The jumps were assumed to be uniformly distributed, which is an important case for the applications.

In Section 2, we derived exact results for a standard Brownian with jumps, while in Section 3 we computed approximately $E[T(x)]$ for important processes, in particular when the continuous part of $X(t)$ is a geometric Brownian motion, which is used very frequently in mathematical finance. The approximate results were either exact or very precise in the interval $[a+\beta_2, b-\beta_1]$, when β_1 and β_2 are both small.

The results presented in Sections 2 and 3 are complementary, because those in Section 2 were valid when $a+\beta_2 \geq b$ and $b-\beta_1 \leq a$, thus when β_1 and β_2 are large enough relative to $b-a$.

When the jump sizes are small, we can use the results obtained in this paper to compute approximately the control that minimizes or maximizes the time spent in the interval (a,b) by the controlled processes $\{X_u(t), t \geq 0\}$ defined by

$$\begin{aligned} X_u(t) &= X_u(0) + \int_0^t b_0\, u[X_u(s)]\,\mathrm{d}s + \int_0^t \mu[X_u(s)]\,\mathrm{d}s \\ &\quad + \int_0^t \sigma[X_u(s)]\,\mathrm{d}B(s) + \sum_{i=1}^{N_1(t)} Y_i + \sum_{i=1}^{N_2(t)} Z_i, \end{aligned} \tag{4.1}$$

where $b_0 \neq 0$ is a constant and $u(\cdot)$ is the control variable; see Lefebvre [8].

Acknowledgment. This research was supported by the Natural Sciences and Engineering Research Council of Canada.

References

1. Abundo, M.: On first-passage times for one-dimensional jump-diffusion processes, *Prob. Math. Statis.* **20** (2000), no. 2, 399–423.
2. Ahlip, R. and Prodan, A.: Pricing FX options in the Heston/CIR jump-diffusion model with log-normal and log-uniform jump amplitudes, *Int. J. Stoch. Anal.* (2015), Article ID 258217, 15 pages. doi: 10.1155/2015/258217
3. Bo, L. and Lefebvre, M.: Mean first passage times of two-dimensional processes with jumps, *Stat. Probabil. Lett.* **81** (2011), no. 8, 1183–1189.
4. Dong, Y. and Han, M.: A hyper-Erlang jump-diffusion process and applications in finance, *J. Syst. Sci. Complex.* **29** (2016), 557–572. doi: 10.1007/s11424-015-3150-0
5. Ghamlouch, H., Grall, A. and Fouladirad, M.: On the use of jump-diffusion process for maintenance decision-making: A first step, *2015 Annual Reliability and Maintainability Symposium (RAMS)*. doi: 10.1109/RAMS.2015.7105099
6. Gihman, I. I. and Skorohod, A. V.: *Stochastic Differential Equations*, Springer, New York-Heidelberg, 1972.
7. Kou, S. G. and Wang, H.: First passage times of a jump diffusion process, *Adv. Appl. Probab.* **35** (2003), no. 2, 504–531.
8. Lefebvre, M.: LQG homing for jump-diffusion processes, *ROMAI J.* **10** (2014), no. 2, 1–6.
9. Merton, R. C.: Option pricing when underlying stock returns are discontinuous, *J. Financ. Econ.* **3** (1976), 125–144.
10. Peng, J. and Liu, Z.: First passage time moments of jump-diffusions with Markovian switching, *Int. J. Stoch. Anal.* (2011), Article ID 501360, 11 pages. doi: 10.1155/2011/501360

Mario Lefebvre: Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal, Québec H3C 3A7, Canada

E-mail address: `mlefebvre@polymtl.ca`